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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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POSITIONS, ORBIT, AND MASS OF PLUTO[1]

By SETH B. NICHOLSON and NICHOLAS U. MAYALL

ABSTRACT

Positions and orbit of Pluto.—Positions of Pluto were obtained from photographs taken by M. L. Humason in 1919 with the 10-inch Cooke triplet and from photographs taken with the 60- and 100-inch reflectors in 1930.

Orbital elements were computed which include the perturbations of the four major planets. The resulting period is 247.6968 years, which corresponds to a mean distance of 39.45743 astronomical units. The time of perihelion passage is 1989 Nov. 6.98 U.T., and the eccentricity 0.24852.

Determination of the mass of Pluto.—The perturbations of Pluto on Neptune were used to determine the mass. A least-squares solution of twenty-two observational equations with the mass of Pluto and the corrections to four of the elements of Newcomb's orbit of Neptune as unknowns gave 0.94 ± 0.25 times the mass of the earth as the mass of Pluto.

I. POSITIONS AND ORBIT OF PLUTO

As soon as it was announced that a trans-Neptunian object had been discovered by Tombaugh at the Lowell Observatory, the records of photographs taken at the Mount Wilson Observatory were examined to see whether any prediscovery positions could be obtained. The only photographs which could show images of the newly discovered object were four of a series begun in December, 1919, by M. L. Humason in a search for trans-Neptunian planets.

This search was stimulated by an article by W. H. Pickering which had just appeared in *Harvard Annals*, **82**, in which he reviewed the previous investigations concerning the existence of a trans-Neptunian planet and gave a new solution of the problem, including the

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 417.

observed deviations of Neptune from its predicted positions. The search covered the position predicted in this paper and also those given by W. H. Pickering in 1909[1] and by Percival Lowell in 1915.[2] The 10-inch Cooke triplet was used with exposures of 2 hours, which showed stars down to about the seventeenth magnitude. The plates were 35.5×43 cm (14×17 in.), with a scale of 2 cm to the degree, thus covering a region 18° in declination by 1^h28^m in right ascension. Three fields were photographed, centered on the ecliptic in right ascension 5^h38^m, 6^h34^m, and 7^h46^m, respectively, and at least three exposures were made on each field during December, 1919, and January, 1920. These plates had been very carefully searched with an improvised blink comparator in a zone 3° wide along the ecliptic, and, although many asteroids and a few variable stars were found, no trans-Neptunian planet was discovered. The search was then extended, but less exhaustively, to a zone about 10° wide, with no success. The search was continued the following year with exposures of 15 minutes at the 60-inch reflector, which covered a region along the ecliptic 1°6 wide and 7° long, centered on the predicted positions. If the planet discovered at the Lowell Observatory had been within a degree and a half of the ecliptic in 1920, it almost certainly would have been found at that time.

The preliminary orbits of Pluto gave the position of the orbit plane with considerable accuracy and indicated that in 1921 the planet was not in the region covered by the 60-inch plates, but was within the field of four of the plates taken with the 10-inch triplet in 1919. The search on these plates was then limited to a narrow region along the orbit plane, but it was not until the region to be examined was still further limited by better orbits that images of the planet were at last found on June 7, 1930. Two ephemerides were used, one by E. C. Bower and F. L. Whipple, which had been received by letter, and one by A. C. D. Crommelin.[3] The latter made use of an observation by Delporte[4] from a plate taken in 1927.

[1] *Harvard Annals*, **61**, 162, 1909.

[2] *Memoirs of the Lowell Observatory*, **1**, No. 1, 1915.

[3] *Circulaire du Bureau Central Astronomique de l'Union Astronomique Internationale*, No. 282, 1930.

[4] *Astronomische Nachrichten*, **238**, 314, 1930.

Four plates of the series taken by Humason showed images of Pluto: two on December 27, 1919, the others on December 28 and 29, respectively. The estimated photographic magnitude of the planet was 15. The positions of Pluto were measured on these plates with reference to six stars, the co-ordinates of which were obtained from the *Catalogue* of the Astronomische Gesellschaft. The mean of these four positions was telegraphed to the Harvard College Observatory.[1]

From the close agreement of the computed and observed positions it was evident that small differential corrections to the orbits on which the ephemerides used in the search were based would suffice to represent the 1919 observations exactly. The observations then available for improving the orbit were those just determined for 1919, the single position by Delporte for 1927, and those made in 1930 since the planet's discovery. It soon became apparent that the declination of the position by Delporte was in error by about 20″, and this observation was therefore discarded. Many observations of the planet had been made between the time of the announcement of its discovery and the end of May, but we decided to base our computations of the orbit on the observations made at Mount Wilson in March and May, 1930, the discovery position made in January, 1930, at the Lowell Observatory, and the 1919 Mount Wilson positions. The following orbit, which best represented these observations, was reported to the Harvard Observatory.[2]

$$
\begin{array}{ll}
T = 1988 \text{ June } 5.5 \text{ U.T.} & q = 29.6 \\
\omega = 111^\circ 46' & e = 0.2575 \\
\Omega = 109\ 22 & P = 251.80 \text{ years} \\
i = 17\ \ 9 &
\end{array}
$$

Soon after the planet's position in 1919 became known, F. E. Ross at the Yerkes Observatory found images of it on plates taken by E. E. Barnard in 1921 and by himself in 1927.[3] There was an apparent discrepancy between the observation of 1921 and the mean of those of 1919 amounting to about 14″ in α. The 1919 plates were

[1] *Harvard Announcement Card*, No. 133, 1930.

[2] *Ibid.*, No. 134, 1930.

[3] *Astronomische Nachrichten*, **239**, 118, 1930.

therefore remeasured with the aid of six faint comparison stars from the Paris zone of the *Astrographic Catalogue*. The images of the planet were almost central in right ascension on the 1919 plates, but were nearly 4° south of the centers of these plates. The eccentric coma of the overexposed images of the A.G. stars might therefore have produced an error in measures of the declination, but should have had no effect on the measures in right ascension. The error due to coma should be nearly eliminated from the positions referred to the fainter stars of the *Astrographic Catalogue*. The new measures gave corrections to the original positions of $+0\overset{s}{.}06$ in α and $-2\overset{''}{.}6$ in δ.

Soon after the 1927 observation by Delporte was shown to be in error, corrections to it were published,[1] amounting to $+0\overset{s}{.}26$ in α and $-15\overset{''}{.}5$ in δ.

The preliminary orbit was next corrected by using these additional observations and the remeasured positions for 1919 and 1927. In the preliminary orbit the mass of the sun alone was used, and the center of the sun was taken as the origin of co-ordinates. The perturbations by the four major planets were therefore computed in rectangular co-ordinates and taken into account in correcting the orbit. Table I gives the corrections which must be applied to the equatorial co-ordinates of Pluto (equator and equinox of 1930.0) derived from an orbit osculating at January 7.0, 1930, to obtain the true co-ordinates. In the meantime, some additional observations had been made at Mount Wilson after Pluto had passed conjunction with the sun, and these were also included in the final calculation of the heliocentric orbit.

It is possible, however, to derive an elliptic orbit which by itself will represent the motion of Pluto more accurately than the heliocentric orbit unless the perturbations are accurately taken into account. This may be done by transferring the origin to the center of mass of the solar system and changing the Gaussian constant k to 0.01721363, which includes the total mass of the system. The motion of Pluto referred to this center is almost exactly elliptic, which explains why the barycentric orbit thus obtained is sufficient to describe the motion with high approximation without the inclusion of perturbations. Geocentric positions computed with the heliocentric

[1] *Ibid.*

TABLE I

PERTURBATIONS BY THE FOUR MAJOR PLANETS

(Unit=0.00001)

Date U.T.	In x	In y	In z
1919 Mar. 6.0	−2668	+1101	+530
1919 Sept. 2.0	2705	1011	493
1920 Feb. 29.0	2723	892	443
1920 Aug. 27.0	2716	751	383
1921 Feb. 23.0	2678	594	315
1921 Aug. 22.0	2607	429	243
1922 Feb. 18.0	2503	261	168
1922 Aug. 17.0	2366	+ 98	96
1923 Feb. 13.0	2198	− 54	+ 27
1923 Aug. 12.0	2003	187	− 34
1924 Feb. 8.0	1785	297	86
1924 Aug. 6.0	1553	380	127
1925 Feb. 2.0	1312	433	155
1925 Aug. 1.0	1072	453	170
1926 Jan. 28.0	841	444	171
1926 July 27.0	628	406	160
1927 Jan. 23.0	441	347	139
1927 July 22.0	286	273	111
1928 Jan. 18.0	167	194	79
1928 July 16.0	83	118	49
1929 Jan. 12.0	31	56	23
1929 July 11.0	6	15	6
1930 Jan. 7.0	0	0	0
1930 July 6.0	4	16	8
1931 Jan. 2.0	− 9	− 61	− 26

elements and perturbations are, however, more accurate than those computed with the barycentric elements alone.

The elements for the two methods of describing the motion are:

	Heliocentric Elements with Mass of Sun	Barycentric Elements with Mass of Solar System	
T	1989 Oct. 2.03 U.T.	1989 Nov. 6.98 U.T.	
ω	113°01′41″.3	113°52′50″.6	1930.0
Ω	109 21 39.4	109 21 43.7	1930.0
i	17 06 58.4	17 08 38.1	1930.0
a	39.60038	39.45743	
e	0.2460861	0.2485200	
P	249.2097 years	247.6968 years	
μ	14″.23833	14″.32530	
q	29.85528	29.65147	
k	0.01720210	0.01721363	

Constants for the Equator 1930.0

$x=r[9.982578]\sin(313°12'55''.1+v)$ $x=r[9.982520]\sin(314°04'18''.8+v)$

$y=r[9.980816]\sin(228\ 15\ 17.7+v)$ $y=r[9.980843]\sin(229\ 06\ 58.1+v)$

$z=r[9.604295]\sin(182\ 03\ 08.0+v)$ $z=r[9.604461]\sin(182\ 50\ 47.8+v)$

Co-ordinates computed by the barycentric elements, which refer to the center of mass of the solar system, may be used to obtain geocentric positions by combining them with co-ordinates of the earth referred to the same origin. Co-ordinates computed with the heliocentric elements must have added to them the perturbations from Table I in order to obtain the true heliocentric co-ordinates. Both sets of elements give the same geocentric position on January 7.0, 1930, and the difference in representation for other dates may be seen from Table II. This table shows the residuals for the various observations used in the computations which correspond to the pre-

TABLE II

Representation of Observations by Different Orbits

Date U.T.	Preliminary Orbit		Preliminary Orbit + Perturbations		Heliocentric Orbit + Perturbations		Barycentric Orbit	
	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$	$\Delta\alpha$	$\Delta\delta$
1919 Dec. 29.0667*	+ 0.″8,	−2.″6	−131.″4,	−3.″0	+1.″1,	+0.″5	+2.″6,	+0.″5
1921 Jan. 29.0896.	+14.4,	−2.1	....		−3.4,	−1.0	−2.8,	−1.2
1927 Jan. 6.2500.	+10.1,	+1.4	....		−2.7,	−3.7	−2.9,	−3.7
1927 Jan. 27.8942.	+13.8,	+5.6	−13.6,	+8.2	+2.6,	+1.9	+2.4,	+1.9
1930 Jan. 23.2274.	0.0,	−2.9	0.0,	−2.9	−0.3,	−3.5	−0.3,	−3.5
1930 Mar. 23.1733*	− 0.1,	−0.3	− 0.1,	−0.3	+0.3,	−0.3	+0.3,	−0.3
1930 May 27.1927*	− 0.2,	+0.3	− 0.1,	+0.3	−0.6,	+0.3	−0.6,	+0.3
1930 Oct. 1.9945*	....		....		+0.1,	+0.3	+0.1,	+0.2

* Normal place.

liminary orbit, to the preliminary orbit with perturbations applied, to the corrected heliocentric orbit with perturbations included, and to the barycentric orbit.

Table III gives the individual observations used in the computations and their residuals obtained from the heliocentric elements by including the perturbations given in Table I. All the Mount Wilson observations in 1930 were derived from photographs taken with the 60-inch reflector, except those of May 27 and June 1, which were taken with the 100-inch reflector.

As shown in Table II, the preliminary orbit indicated a discrepancy of about 14″ between the position of 1921 and that of 1919. The corrected orbit represents all the observations of 1919 and 1921 within the errors of measurement inherent in plates of such small scale. It is evident, therefore, that the apparent discrepancy was

due to the perturbations of the major planets, and not to errors of observation.

No ephemeris has been computed from the final elements given here, since they differ so little from those derived by Bower and Whipple[1] that the ephemeris computed by them is entirely sufficient. After these elements were obtained, an observation from a plate taken in 1914 was published by Wolf.[2] The representation of this

TABLE III

OBSERVATIONS AND THEIR REPRESENTATION BY HELIOCENTRIC ORBIT WITH PERTURBATIONS INCLUDED

Date U.T.	α(1930.0)	δ(1930.0)	Observer	$\Delta\alpha$	$\Delta\delta$
1919 Dec. 28.1861..	$6^h29^m08^s.53$	$+19°21'47''.0$	Humason	$+2''.5$	$+0''.1$
1919 Dec. 28.2861..	6 29 07.68	19 21 49.0	Humason	−2.5	+1.4
1919 Dec. 29.4285..	6 29 02.13	19 21 56.8	Humason	+3.1	+1.2
1919 Dec. 30.3660..	6 28 57.14	19 22 01.5	Humason	+1.2	−0.7
1921 Jan. 29.0896..	6 31 22.14	19 43 13.8	Barnard	−3.4	−1.0
1927 Jan. 6.2500..	7 04 03.17	21 13 03.1	Ross	−2.7	−3.7
1927 Jan. 27.8942..	7 02 10.69	21 17 29.3	Delporte	+2.6	+1.9
1930 Jan. 23.2274..	7 18 56.37	21 57 40.1	Tombaugh	−0.3	−3.5
1930 Feb. 23.1944..	7 16 36.39	22 04 06.8	Tombaugh	−1.1	−1.2
1930 Mar. 23.1733..	7 15 33.83	22 07 55.4	Mayall	+0.4	−0.1
1930 Mar. 24.1458..	7 15 33.02	22 08 00.7	Mayall	+0.2	−0.2
1930 Mar. 28.1497..	7 15 30.80	22 08 20.7	Mayall	+0.4	−0.5
1930 Apr. 22.1602..	7 15 54.82	22 09 18.2	Mayall	−0.6	+0.5
1930 May 22.1719..	7 17 43.42	22 07 56.2	Mayall	−0.9	+0.3
1930 May 27.1927..	7 18 08.72	22 07 27.8	Pease	−0.5	−0.3
1930 June 1.1809..	7 18 35.51	22 06 58.1	van Maanen	−0.3	+1.0
1930 Oct. 1.4906..	7 29 57.22	21 51 43.3	Mayall	+0.4	+0.4
1930 Oct. 2.4984..	7 29 59.35	+21 51 42.3	Nicholson	−0.2	+0.2

observation by the barycentric elements is $\Delta\alpha = +9''.9$, $\Delta\delta = +0''.8$. The residuals from the orbit of Bower and Whipple are $\Delta\alpha = -112''.5$, $\Delta\delta = -25''.2$.[3] These indicate the order of the corrections due to perturbations which should be applied to their search ephemeris.

II. DETERMINATION OF THE MASS OF PLUTO

The mass of Pluto may be determined from its perturbations on Neptune and Uranus. A recent paper by J. Jackson[4] on "The Orbit

[1] *Lick Observatory Bulletin*, **15**, 35 (No. 427), 1930.

[2] *Nature*, **126**, 485, 1930.

[3] *Harvard Announcement Card*, No. 142, 1930.

[4] *Monthly Notices of the Royal Astronomical Society*, **90**, 728, 1930.

of Neptune" gives a summary of the differences between the observed longitudes of that planet and those computed from Newcomb's elements. With this convenient summary available a solution for the mass of Pluto from its perturbations on Neptune was readily made. Normal equations were formed from the twenty-two observation equations which Jackson used, with the mass of Pluto added as a fifth unknown. These equations then read

$$\Delta\lambda = \alpha + \beta t + \gamma \cos g + \delta \sin g + m p_\lambda \ ,$$

where t is the time in centuries from 1870.0, g the mean anomaly of Neptune, m the mass of Pluto in units of the earth's mass, and p_λ the perturbation in λ which Pluto would have produced on Neptune if its mass were unity. The origin of time for both t and p_λ was chosen as 1870.0, since that was near the middle of the series of observations on which Newcomb's orbit of Neptune was based. The values of p_λ are given in Table IV. Corresponding values for p_β, the perturbation in β, are also included in Table IV. The magnitude of the perturbations in latitude at the present time is such that their inclusion might have improved the determination of the mass of Pluto, but since no homogeneous series of observations of the latitude of Neptune was immediately available, they were not used.

A least-squares solution of the twenty-two observational equations, with equal weights, gave

$$m = 0.94 \pm 0.25$$

$$\alpha = -0''.72 \qquad \gamma = +0''.63$$

$$\beta = -0''.18 \qquad \delta = -0''.25$$

Since the observation equation for 1795 is based on two prediscovery positions by Lalande, while the others are the means of several observations made during four-year intervals, there seems to be sufficient reason for assigning less weight to the equation depending on Lalande's observations.

A second solution was therefore made, with zero weight assigned to the doubtful equation.

The solution of the normal equations based on the remaining 21 observation equations is nearly indeterminate. Values of m ranging

from 0 to 1.5, with values of the other four unknowns to correspond, represent the observation equations almost equally well. The residual for the mean of Lalande's observations is $-6''.2$ with $m=0$, $0''.0$ with $m=0.9$, and $+3''.5$ with $m=1.5$.

The solution for the mass of Pluto from observations of the longitude of Neptune depends, therefore, on the observations of Lalande and indicates that if the mass of Pluto is less than 0.6 or greater than 1.3, Lalande's observations are in error by more than $2''$, or else the

TABLE IV

PERTURBATIONS OF PLUTO ON NEPTUNE
ASSUMED MASS OF PLUTO = 1.0 EARTH

Date	p_λ	p_β	Date	p_λ	p_β
1796	$-0''.82$	$-0''.02$	1864	$+0''.01$	$0''.00$
1800	$-$ .69	$+$ .09	1868	$+$ .00	.00
1804	$-$ .57	$+$ .16	1872	$+$ .00	.00
1808	$-$ .45	$+$ .19	1876	$+$ .01	.00
1812	$-$ .34	$+$ .18	1880	$+$ .03	.00
1816	$-$ .24	$+$ .16	1884	$+$ .05	$+$ .01
1820	$-$ .14	$+$ .11	1888	$+$ .07	$+$ .03
1824	$-$ .07	$+$ .06	1892	$+$ .08	$+$ .04
1828	$-$ .01	$+$ .01	1896	$+$ .05	$+$ .04
1832	$+$ .04	$-$ .04	1900	$-$ .02	$+$ .01
1836	$+$ .06	$-$ .06	1904	$-$ .15	$-$ .08
1840	$+$ .08	$-$ .07	1908	$-$ .33	$-$ .20
1844	$+$ .08	$-$ .07	1912	$-$ .58	$-$ .35
1848	$+$ .08	$-$ .07	1916	-0.90	$-$ .54
1852	$+$ .06	$-$ .06	1920	-1.23	$-$ .77
1856	$+$ .04	$-$ .04	1924	-1.57	-0.96
1860	$+0.02$	-0.01	1928	-1.93	-1.09

residuals of the other observations are larger than those given by this solution. Such a dependence on the early observations in problems of this sort is to be expected, according to E. W. Brown.[1]

The corrections to Newcomb's orbit of Neptune obtained from all twenty-two equations are appreciably smaller than those derived by Jackson on the assumption of no unknown perturbing masses. The residuals of each observation equation given by Newcomb's orbit, by the solution which includes Lalande's observations, and by Jackson's orbit of Neptune are given in Table V. The third column of the table gives the corrections due to the mass factor, which are obtained by multiplying p_λ from Table IV by 0.94.

[1] *Proceedings of the National Academy of Sciences*, **16**, 364, 1930.

In his paper Jackson says:

It may be worth pointing out that the existence of a disturbing body would be shown by the residuals running through two complete cycles in the interval covered by the observations. When we analyse the residuals in longitude to correct the elements we really have four disposable constants, and the natural effect of attempting to fit the observed and computed longitudes (if a disturbing body is present) is to make the residuals vanish four times in the interval covered by the observations. This is very clearly shown in the residuals Leverrier found after making the best fit for Uranus, and the fact that a similar series of residuals is not found in the case of Neptune indicates that the effect of such outside bodies as may exist does not exceed the error of observation.[1]

TABLE V

RESIDUALS FOR NEPTUNE IN LONGITUDE

Date	O−C Newcomb	Corr. for Pluto	O−C Jackson	O−C All Equations	Fourth Col. − Fifth Col.
1795	−2″05	−0″80	−0″12	−0″01	−0″11
1848	−0.27	+ .08	+ .27	− .09	+ .36
1852	+ .26	+ .06	+ .61	+ .39	+ .22
1856	+ .07	+ .04	+ .27	+ .15	+ .12
1860	− .52	+ .02	− .45	− .46	+ .01
1864	− .37	+ .01	− .41	− .32	− .09
1868	+ .12	.00	.00	+ .16	− .16
1872	− .30	.00	− .46	− .26	− .20
1876	+ .20	+ .01	+ .04	+ .25	− .21
1880	+ .32	+ .03	+ .19	+ .39	− .20
1884	− .36	+ .05	− .40	− .26	− .14
1888	− .30	+ .07	− .23	− .17	− .06
1892	+ .25	+ .08	+ .48	+ .45	+ .03
1896	− .28	+ .05	+ .14	+ .04	+ .10
1900	− .75	− .02	− .11	− .28	+ .17
1904	−0.72	− .14	+ .18	− .02	+ .20
1908	−1.03	− .31	+ .16	− .05	+ .21
1912	−1.28	− .55	+ .20	+ .06	+ .14
1916	−1.71	−0.85	+ .09	+ .03	+ .06
1920	−2.29	−1.16	− .16	− .13	− .03
1924	−2.59	−1.48	− .14	− .00	− .14
1928	−2.92	−1.81	−0.14	+0.10	−0.24
[vv]	....	....	1.80	1.22	....

In order to exhibit this effect the residuals from Jackson's orbit are plotted in the upper part of Figure 1. The continuous curve is the difference between Jackson's corrections to Newcomb's residuals and the corrections including a mass factor of 0.94 for Pluto. These

[1] *Ibid.*

differences are given in the sixth column of Table V and may be obtained by subtracting the fifth column from the fourth. They may be thought of as a smooth curve drawn through Jackson's residuals, which are comparable with the residuals of Uranus found by Leverrier before the effect of Neptune was taken into account. These residuals of Uranus, which are also given in Jackson's paper, have a maximum range of 31″ compared with the range of 0″.60 found in the smoothed curve for Neptune. The probable error of the mean

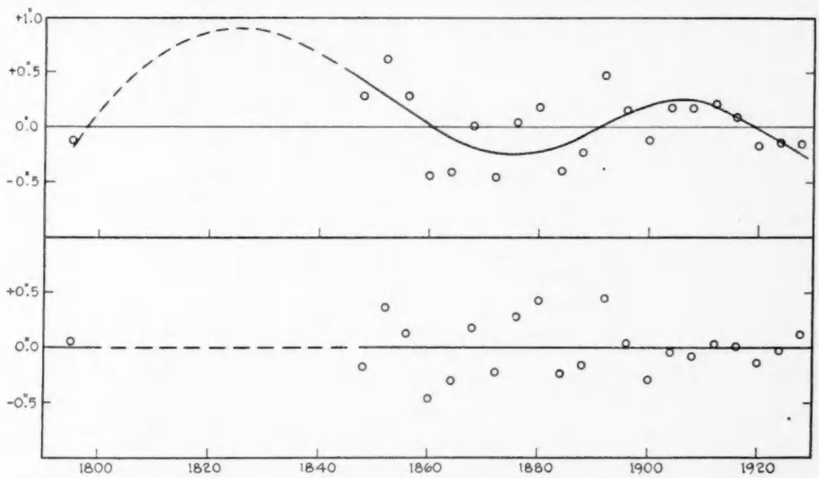

FIG. 1.—The residuals of Neptune in longitude. Above, those derived by Jackson with no perturbing masses assumed; below, those remaining after correction for perturbations by Pluto based on a mass of 0.94 (earth = 1).

of the observations of Neptune combined in four-year intervals, as used in these solutions, is ±0″.18. The residuals of Neptune when a mass of 0.94 for Pluto is introduced are given in the fifth column of Table V and are plotted in the lower part of Figure 1.

The mass of Pluto can be determined more accurately when its perturbations on Neptune have reached a maximum and when the perturbations in latitude are considered. Although the maximum perturbations by Pluto on Uranus are smaller than those produced on Neptune, they may yield a more reliable value of Pluto's mass because at the present time Uranus has been observed at more conjunctions with Pluto than has Neptune. The photographic magni-

tude of Pluto has been estimated from 15 to 16 by various observers. A single comparison with the North Polar Sequence made with the 60-inch reflector gave 15.5. Preliminary observations with a yellow color-screen indicate that the color-index is at least one magnitude. The visual magnitude, therefore, probably lies between 14.0 and 14.5. If Pluto has a density equal to that of the earth and if it reflects like the moon, a visual magnitude of 14.0 would correspond to a mass two-thirds that of the earth; for a magnitude of 14.5 the mass would be one-third that of the earth. If the mass of Pluto is more than two-thirds that of the earth, the density must be higher or the albedo lower than any yet determined in the solar system. Although Pluto may differ radically from other planets in these respects, a consideration of its magnitude favors the lower limit of the mass derived here. Until further evidence becomes available, it may be accepted as probable that the mass of Pluto is of the order of two-thirds that of the earth.

NOTE.—While the preceding article was in press, a paper by F. Zagar[1] was received. His results for the orbit of Pluto, after taking into account the perturbations of the four major planets, are in substantial agreement with those given here.

CARNEGIE INSTITUTION OF WASHINGTON
MOUNT WILSON OBSERVATORY
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[1] *Astronomische Nachrichten*, **240**, 335, 1930.

TEMPERATURE CLASSIFICATION OF THE STRONGER LINES OF COLUMBIUM, WITH PRELIMINARY NOTES ON THEIR HYPERFINE STRUCTURE[1]

By ARTHUR S. KING

ABSTRACT

The paper gives the temperature class, the ionization, and a preliminary discussion of the hyperfine structure of 646 of the stronger lines of columbium between λ 3100 and λ 6900. The results were obtained by comparing electric-furnace spectra at temperatures of about 2500° and 2900° C with the spectra of the arc and spark. Approximately 200 lines appear in the furnace spectrum, but many neutral and all of the ionized lines require higher excitation. Much variation in response to excitation was found for lines in both groups.

Of special interest is the large proportion of columbium lines having hyperfine structure. Of the lines listed, about 40 per cent are evidently complex. The patterns, for the most part unresolved, appear to consist usually of two, four, or six components, with some still more complex. As the wave-length increases, the spacing of hyperfine patterns becomes in general wider. Lines of complex structure occur in all temperature classes.

The data presented in this paper cover the stronger lines emitted by the columbium arc from λ 3100 to λ 6900. Of these, the proportion appearing in the electric-furnace spectrum is smaller than for other elements which have been studied, probably in large measure because of the high vaporization-point of the metal. The lines of the ionized atom, selected by a comparison of arc and spark spectra, are beyond the reach of the furnace excitation, and only those which attain considerable strength in the arc are included in the present discussion. These and the neutral lines of sufficient strength to be expected in the furnace spectrum were selected from a very large preliminary list. All lines below intensity 8 were omitted.

To the difficulty arising from the faintness of the furnace spectrum is added that caused by the masking of many columbium lines by the carbon bands (from the graphite tube), which are strong at temperatures required for the vaporization of the metal. The results give, however, the behavior of the more sensitive lines in both the ionized and neutral spectra, the degree of excitation required for the latter being indicated by the furnace intensities.

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 416.

EXPERIMENTAL METHOD

The spectrograms were made with the 15-foot concave-grating spectrograph, the second-order spectrum (scale, 1 mm = 1.86 A) being used to the violet of λ 5400. The graphite tube of the vacuum furnace was charged with powdered columbium and heated, for the high-temperature stage, to 2800° or 2900° C. Another series of furnace spectrograms made at 2400°–2500° was used to determine the persistence of the furnace lines.

In addition, a large collection of arc and spark spectrograms was made, to serve for the selection of ionized lines and for measurement of the columbium wave-lengths from iron standards. The electrodes used to obtain these spectra were of highly purified columbium procured from Dr. C. W. Balke, of the Fansteel Products Company, by Dr. W. F. Meggers, of the Bureau of Standards, and kindly loaned to the writer.

EXPLANATION OF THE TABLE

The wave-lengths in the first column of Table I are on the international system, converted from the values of Exner and Haschek,[1] with some wave-lengths, usually of close doublets, measured by the writer. The second column gives the estimated arc intensities, none of which is less than 8. Partial reversal, observed for a few arc lines, is indicated by *r*. The furnace intensities in the third column are for the high-temperature stage, 2800°–2900° C. The frequent question marks in this column indicate uncertainties arising from blends with foreign lines, usually those of the carbon bands. The temperature classes in the final column are based on the strength of the lines in the arc, in the high-temperature furnace, and in the furnace at about 2400° C, data for the last not being entered in the table. Lines of the ionized spectrum, none of which appears in the furnace, are placed in class V E. An asterisk (*) after a wave-length refers to a note at the end of the table.

The probable number of hyperfine components, when the line is evidently not single, is indicated by an Arabic numeral following the class number. The significance of these component-numbers is given in the discussion of this feature of the spectrum.

[1] *Spektren der Elemente*, **2**, 1911.

TABLE I

TEMPERATURE CLASSIFICATION OF COLUMBIUM LINES

λ	Intensities		Class No. Comps.
	Arc	Fur.	
3094.21	300†	....	V E, 6
3096.50	10	....	V
3099.20	15	....	V E
3111.47	20	....	V
3116.34	10	....	V
3122.66	10	....	V
3127.53	30	....	V E, 4
3130.81	200†	....	V E, 4
3133.01	10	....	V
3136.97	8	....	V
3145.40	50†	....	V E
3151.86	30	2	IV
3163.39	150†	....	V E, 2
3172.54	8	....	V
3175.80*	10 8	....	V E, 3 V E
3180.29	40†	....	V E
3186.54	10	....	V, 2
3187.46	40	3	IV
3189.24	10	....	V E, 4
3191.10	100	....	V E, 6
3191.44	15	....	V E
3194.97	150†	....	V E
3200.52	10	1	IV
3206.32	60†	....	V E
3210.16	10	....	V
3215.24	8	....	V
3215.59	80†	....	V E, 4
3217.27	30	....	V
3217.83	20	....	V
3220.91	8	....	V
3221.13	12	....	V
3223.32	10	....	V E
3225.46	200†	....	V E, 6
3229.55	20	....	V E
3236.44	80†	....	V E, 2
3238.06	8	....	V E, 6
3246.77	15	....	V
3247.47	12	....	V E, 2
3248.94	10	....	V E
3249.50	50	4	IV
3251.55	8	....	V
3251.66	30	1	IV
3254.03	60†	....	V E, 3
3260.13	20	....	V
3260.56	20	....	V E, 4
3261.89	15	....	V, 2
3263.36	10	....	V E
3264.59	50	2	IV
3267.05	12	....	V
3270.44	20	1	IV, 2
3270.76	25	2	IV.....
3271.98*	10	....	V
3272.07*	30	4	IV
3277.65	50	4	IV
3279.24	8	....	V E, 4
3283.45	15	....	V E, 2
3285.66	80	6	IV
3287.59	60	3	IV
3287.91	50	....	V
3291.91*	8	....	V
3292.00*	12	....	V E
3296.03*	80	6	IV
3299.62	30	1	IV
3304.86	15	....	V
3308.05	40	2	IV
3310.45	20	1	IV
3311.37	8	....	V
3312.61	150	10	IV
3315.20	50	....	V
3318.96	60	4	IV, 2
3319.25*	25	....	V E V
3319.57	15	....	V E
3326.61*	60	1	IV
3329.37	30	....	V, 2
3332.16	30	1	IV
3335.40	10	....	V, 4
3336.31	10	....	V
3341.58	10	....	V E
3341.95	300	30	III
3343.68	150	20	III
3346.92	15	....	V
3349.04	200	25	III
3349.51*	20	....	V, 4
3352.27	10	....	V
3352.59	8	....	V, 2
3353.52	12	....	V E
3354.69	100	4	III
3355.41	8	....	V
3357.02	30	....	V
3358.39	300	25	III
3365.58	20	....	V E
3366.98	80	1	IV
3367.38	20	....	V
3369.21*	10	....	V E, 2

* See notes at end of table.

† Reversed in spark.

TABLE I—*Continued*

λ	Intensities Arc	Intensities Fur.	Class No. Comps.	λ	Intensities Arc	Intensities Fur.	Class No. Comps.
3371.33	60	2	IV	3456.52	10	....	V
3372.12	10	....	V	3457.79	10	....	V
3372.58	8	....	V E, 4	3458.94	10	....	V
3374.92	100	2	IV	3459.68	20	....	V, 2
3376.33	10	....	IV	3462.64	15	....	V
3376.73	12	....	IV, 2	3463.03	8	....	V
3380.06	20	....	V	3463.68*	8	....	V
3380.45	40	4	III	3463.80*	25	....	V, 4
3383.75	8	....	V, 2	3465.85*	50	2?	IV
3384.66	10	....	V	3467.46	8	....	V, 2
3385.66	8	....	V	3469.43	15	....	V
3386.24	20	....	V E	3473.06	20	....	V, 4
3387.68	10	....	V	3475.61	8	....	V
3387.79	15	....	V	3478.70	40	....	V, 2
3390.64	20	....	V, 4	3478.81	8	....	V E, 2
3392.34	150	6	IV, 2	3479.55	20	....	V E
3395.94	50	2	IV	3481.06	10	....	V
3398.24	12	....	V, 2	3484.05	12	....	V E, 2
3399.41	30	1	IV, 2	3489.07	10	....	V E
3399.71	15	....	V E, 2	3491.05	50	1	IV, 2
3399.97	8	....	V	3491.48	10	....	V
3403.02	10	....	V, 4	3497.78*	40	2?	IV
3403.73	10	....	V	3498.61	{40	4	IV
3405.39	60	....	V, 2		{30	....	V
3406.11	40	....	V	3503.18	15	....	V
3406.60	10	....	V	3507.94	80	8	III
3408.35	60	4	IV	3510.26*	15	....	V E, 2
3408.67	30	....	V E	3511.16	15	....	V, 2
3409.16	20	....	V E	3515.40	15	....	V E, 2
3409.89	12	....	V	3516.84	15	....	V
3412.90	30	....	V E	3518.16	8	....	V
3414.03	15	....	V	3520.05	40	2	IV
3415.96	15	....	V, 4	3520.71	12	....	V
3417.86	10	....	V	3525.22	20	....	V, 4
3420.62	10	....	V E	3533.65	60	4	IV
3423.76	40	1	IV	3535.29	500	20	III, 2
3425.44	30	....	V E, 2	3537.51	200	15?	III
3425.85	20	....	V, 4	3537.65	10	....	V E
3426.56	25	....	V E, 2	3539.62	10	....	V
3427.47	60	4	IV	3540.99	30	....	V E
3428.77	10	....	V, 4	3542.97	10	....	V
3429.06	20	....	V	3543.91*	10	....	V
3431.97	8	....	V	3544.01*	60	?	IV?
3432.42	15	....	V, 4	3544.65	50	2	IV
3432.72	20	....	V E	3548.11	10	....	V E?
3433.11*	15	....	V	3550.44	50	?	IV?
3436.96*	8	....	V, 2	3554.52*	40	?	IV?, 2
3439.91	15	....	V E	3554.65*	80	6	III
3440.62	40	....	V E	3559.11	8	....	V
3442.65	10	....	V	3563.49*	100	3	IV
3445.65	20	....	V, 4	3563.60*	80	2	IV
3452.35	15	....	V, 4	3568.75	12	....	V

TABLE I—*Continued*

λ	Intensities Arc	Intensities Fur.	Class No. Comps.	λ	Intensities Arc	Intensities Fur.	Class No. Comps.
3569.46......	50	....	V	3717.51......	12	....	V, 2
3575.86......	200	8	III	3725.21......	8	....	V, 4
3577.71......	40	....	V	3726.26......	250	20	III
3580.28......	500r	30?	III?	3738.47......	20	....	V
3584.96......	100	?	IV?	3739.86......	300r	30	III
3589.09......	100	?	IV?	3740.73*.....	10	....	V E
3589.35......	80	....	V, 4	3740.84*.....	50	3	III
3593.96......	80	6	III	3741.83......	20	2	IV, 2
3599.27......	10	....	V, 2	3742.46......	200r	20	III
3599.62......	15	....	V	3744.04......	20	2	IV, 2
3602.56......	60	4	III	3746.95......	10	....	V
3615.47......	12	....	V, 4	3748.56......	15	....	V
3617.67......	8	....	V	3753.20......	30	1	IV
3618.40......	8	....	V	3755.77......	15	....	V, 2
3618.86......	10	....	V	3759.59......	200r	15	III
3619.46......	30	....	V E	3763.48......	50	4	III
3619.68......	10	....	V E, 2	3764.10......	15	....	V
3621.01......	30	....	V	3765.08......	25	....	V, 2
3625.15......	8	....	V	3766.15*.....	20	....	V, 2
3625.70......	8	....	V	3769.16......	15	....	V
3633.71......	15	....	V	3770.89*.....	10	....	V
3636.95......	15	....	V, 4	3771.85......	30	....	V
3637.52......	20	....	V	3775.46......	10	....	V
3637.84*....	25	....	V, 4	3781.07......	60	5	III
3638.75......	15	....	V, 2	3786.21......	8	....	V
3639.30......	40	....	V, 4	3787.12......	150	3?	IV?
3640.65......	30	....	V	3790.16......	200r	10	III
3644.91......	15	....	V	3791.29......	300r	15	III
3649.84......	80	3	III	3795.56......	15	....	V
3650.77......	30	....	V, 4	3796.46*.....	10	....	V, 4
3651.17......	50	....	V E	3798.15......	300r	10	III, 2
3659.59......	15	....	V E, 2	3800.96......	20	....	V, 2
3660.37......	100	2	IV, 2	3801.27*.....	10	....	V
3664.71......	80	10	III	3802.99......	400r	2?	IV?
3668.61......	15	....	V	3803.92......	100	2?	IV?
3669.01......	40	....	V	3804.75......	40	....	V E, 2
3674.77*.....	50	....	V, 6	3806.17......	10	....	V
3687.95......	15	....	V E, 6	3810.50......	80	....	V, 6
3693.36......	15	....	V	3811.03......	60	4?	III?
3694.66......	10	....	V, 2	3815.47......	60	?	IV?, 2
3697.39......	15	....	V	3818.83......	15	....	V E
3697.86......	200	20	III	3819.18......	20	....	V, 2
3703.16*.....	8	....	V E, 4	3824.87......	100	2?	IV?
3703.90......	8	....	V, 2	3831.85......	12	....	V E
3704.15*.....	15	....	V E, 4	3835.17......	30	?	IV?
3709.43*.....	20	....	V	3836.47......	10	....	V
3711.36......	50	....	V, 2	3842.70......	8	....	V
3713.07......	400r	30	III	3845.96......	30	....	V
3713.79*.....	15	....	V	3858.93......	40	?	V?
3716.19*.....	10	....	V, 4	3862.93*.....	10	....	V, 4
3717.02......	{20	....	V, 2	3863.38......	40	?	V?, 2
	20	....	V E	3867.91......	30	?	V?, 4

TABLE I—*Continued*

λ	Intensities		Class No. Comps.	λ	Intensities		Class No. Comps.
	Arc	Fur.			Arc	Fur.	
3871.19*.....	8	....	V, 4	4001.12......	8	....	V, 4
3875.78......	10	....	V	4008.27......	20	....	V
3876.95......	8	....	V, 2	4009.69	20	....	V
3877.52......	50	....	V, 2	4013.26......	20	....	V
3878.84*.....	40	?	V?	4017.51......	10	....	V
3878.95*.....	15	?	V	4023.14......	8	....	V
3883.14*.....	100	?	V?	4027.30......	8	....	V
3885.45......	100	3	IV, 2	4027.95......	8	....	V, 2
3885.73......	100	1	IV	4032.57......	200	10	IV, 2
3886.08......	20	....	V	4033.19......	50	....	V
3891.37......	60	2	IV	4039.07......	12	....	V, 2
3893.75......	30	3	IV	4039.54......	60	....	V, 6
3894.09......	40	1	IV	4044.10......	20	....	V, 2
3895.91......	10	....	V	4044.71......	10	....	V
3898.56......	8	....	V	4049.75......	12	1?	IV?
3899.24......	8	....	V, 4	4051.53......	20	....	V
3904.18......	15	....	V, 2	4058.99......	2000	100	III, 6
3906.91......	8	....	V	4059.50......	30	....	V
3908.98......	30	2	IV	4060.82......	30	1?	IV?, 2
3914.73......	150	3	IV	4067.17......	12	....	V
3919.00......	8	....	V	4068.26......	20	....	V, 4
3919.15......	8	....	V, 2	4070.98......	10	....	V
3920.24......	100	1	IV, 4	4079.73......	1000	50	III, 4
3922.32*.....	10	....	V	4084.19......	8	....	V
3925.00......	40	....	V, 2	4084.87......	40	1	IV
3929.31......	30	1	IV, 2	4090.16......	10	2	III
3934.16......	10	....	V, 6	4099.10......	20	1	IV, 2
3934.42......	15	....	V	4100.40......	80	3	IV, 2
3935.45......	8	....	V, 4	4100.99......	600	30	III, 2
3936.46......	8	....	V	4112.15......	10	....	V, 4
3937.54......	150	4	IV, 2	4113.95......	20	....	V
3937.99......	15	....	V	4116.91......	40	5	III, 2
3941.31......	30	....	V, 2	4122.81......	8	....	V
3943.69......	60	4	IV, 2	4123.86......	400	15	III
3947.50......	10	....	V, 2	4125.24......	8	....	V, 4
3955.66......	15	....	V	4129.45......	100	4	IV, 4
3959.36......	8	....	V, 2	4129.97......	125	....	V, 4
3961.02......	8	....	V, 6	4134.63......	20	1	IV
3965.70......	40	1	IV	4137.15......	200	8	III, 2
3966.14*.....	30	....	V	4139.43......	100	3	IV, 2
3966.29*.....	200	....	V E	4139.74......	300	10	IV, 6
3970.70......	8	....	V	4143.22......	60	1	IV, 2
3971.93......	{20	....	V	4147.19......	10	?	IV?, 2
	{15	....	V	4150.17......	100	....	V
3972.54......	30	....	V, 2	4152.06......	10	....	V, 2
3976.64......	10	....	V, 2	4152.65......	600	20	III, 4
3977.95......	12	....	V, 4	4158.02......	10	....	V
3978.76......	10	....	V	4163.48......	50	....	V
3979.38......	8	....	V	4163.64......	400	10	III
3980.53......	60	1	IV, 4	4164.65......	400	10	III
3991.68......	10	....	V, 4	4168.14......	200	10	IV, 4
3999.17......	40	....	V	4169.58......	20	....	V, 2

TABLE I—*Continued*

λ	Intensities		Class No. Comps.	λ	Intensities		Class No. Comps.
	Arc	Fur.			Arc	Fur.	
4184.44	60	2	IV, 2	4342.83	15	....	V
4186.08	8	....	V	4348.64	15	....	V, 6
4190.67	15	....	V, 2	4349.03	15	....	V
4190.93	100	2	IV, 6	4351.64	40	?	IV?
4192.07	60	2	IV, 4	4359.81	12	....	V, 6
4195.13	60	2?	IV?	4368.46	30	1?	IV?, 2
4195.65	10	....	V	4377.96	10	....	V, 2
4198.46	{8	....	V, 4	4388.37	15	....	V
	{30	1	IV	4392.74	15	....	V
4201.54	40	....	V	4410.24	50	3	IV
4205.34	100	4	IV, 2	4411.53	8	....	V
4208.13	15	....	V	4419.45	40	2	IV
4212.03	8	....	V	4419.84	8	....	V
4212.51	8	....	V, 2	4420.45	8	....	V
4214.75	50	....	V, 6	4420.64	20	....	V, 4
4217.98	125	4	IV, 4	4426.67	15	....	V, 2
4229.15	60	....	V, 6	4429.45	10	....	V, 2
4229.82	20	....	V	4437.25	50	....	V, 6
4230.32	10	....	V	4441.82	8	....	V, 4
4231.95	20	1	IV	4446.21	8	....	V, 2
4242.64	8	....	V, 4	4447.24	80	4	IV
4246.28	10	....	V, 2	4456.81	30	3	IV
4249.46	12	1	IV	4457.41	30	?	IV?, 2
4253.05	30	....	V, 6	4458.10	8	....	V
4253.74	20	....	V, 4	4460.19	8	....	V, 4
4254.75	20	....	V	4460.45	10	....	V, 2
4255.51	40	....	V, 4	4464.15	10	2	IV
4261.72	15	....	V, 6	4469.73	20	1	IV, 2
4262.15	100	6	IV	4471.33	20	....	V, 4
4266.06	30	1	IV	4472.56	20	2	IV, 4
4268.66	8	....	V	4499.84	10	....	V, 2
4270.71	20	....	V, 6	4503.07	40	5	III
4277.47	15	....	V	4508.44	10	....	V
4280.64	25	....	V, 2	4511.12	10	....	V, 2
4286.20	10	....	V	4523.45	150	10	III, 2
4286.99	50	4	IV	4524.14	10	....	V
4289.47	25	....	V	4546.83	100	15	III
4291.26	20	....	V, 2	4553.84	15	....	V
4292.04	15	....	V, 4	4564.56	50	....	V
4292.55	25	....	V, 6	4573.13	200	10	III, 2
4295.55	10	....	V	4574.85	10	....	V, 4
4296.24	12	....	V, 4	4581.68	80	....	V, 2
4299.65	100	....	V	4582.29	10	....	V
4301.19	100	....	V	4600.22	20	....	V
4309.57	10	....	V	4606.79	300	15?	III, 4
4311.33*	30	....	V, 2	4616.13	40	....	V, 2
4312.45	15	....	V	4630.13	100	2?	IV?
4326.38	40	....	V, 6	4648.93	100	?	IV?, 2
4327.42	10	....	V, 2	4649.25	20	....	V, 6
4328.44	8	....	V, 4	4663.82	100	2?	IV?
4329.75	8	....	V, 4	4666.21	50	?	IV?
4331.44	50	....	V	4667.20	30	....	V

TABLE I—*Continued*

λ	Intensities Arc	Intensities Fur.	Class No. Comps.	λ	Intensities Arc	Intensities Fur.	Class No. Comps.
4672.08	200	?	V?, 6	5193.04	30	....	V, 4
4675.36	150	?	V?, 4	5195.84	25	6	IV
4678.43	{6, 8}	....	V, 2	5203.18	8	....	V
				5219.09	10	....	V, 6
4685.11	50	?	V?	5225.14	8	....	V
4694.46	8	....	V, 4	5232.83	25	....	V
4695.42	15	....	V, 6	5251.65	15	....	V, 6
4697.43	20	....	V, 2	5253.93	15	....	V
4706.11	40	....	V, 6	5271.53	60	5	IV, 2
4708.23	60	?	IV?, 2	5276.13*	30	....	V, 6
4713.45	30	....	V, 2	5318.61	50	20	III
4733.43	15	....	V, 2	5319.47	8	....	V, 2
4733.85	30	....	V, 6	5334.85	15	1	IV, 4
4743.82	8	....	V	5340.80	10	....	V
4744.61	20	....	V	5344.20	200	40?	III, 4
4749.60*	60	....	V, 6	5350.78	100	15	III, 2
4766.81*	10	....	V, 4	5381.33	10	5?	III?
4809.32	12	....	V, 2	5411.25	8	....	V
4810.57*	30	....	V, 6	5422.43	20	2	IV
4816.33	50	2	IV, 4	5431.28	12	....	V, 2
4829.25	15	....	V	5437.30	80	8	III
4833.34	30	5?	III?	5481.01	10	....	V
4838.02	8	....	V, 6	5504.61	20	8?	III, 2
4845.15	10	....	V	5512.84	12	....	V, 6
4848.27*	60	....	V, 6	5523.59	30	?	IV?
4868.98	12	....	V	5551.39	80	....	V
4890.75	20	1	IV	5562.98	8	....	V
4900.78	10	5	III	5576.13	12	....	V, 4
4904.53	15	2	IV	5578.28	10	....	V
4910.92	15	3	IV, 2	5586.97	40	2	IV
4928.96	8	?	IV?	5603.51	30	....	V
4965.35	20	....	V, 6	5629.17	40	....	V
4967.81	20	2	IV, 6	5642.10	100	4	III
4973.13	15	....	V	5664.68	150	8	III
4988.98	40	3	IV, 6	5665.59	100	3	III
4997.85	8	....	V, 2	5671.06*	40	....	V, 6
5000.94	10	....	V, 4	5671.88	40	2	IV
5017.73	40	?	IV?, 2	5706.49	40	2?	IV?
5026.35	15	....	V, 2	5716.38	30	....	V
5039.05	40	3	IV, 2	5729.18	100	10	III
5058.01	30	?	IV?	5751.44	30	....	V
5065.24	20	?	IV?	5760.37	80	1	IV
5078.94	200	15	IV	5765.00	30	....	V
5095.31	80	?	IV?, 2	5776.08*	12	....	V, 6
5100.16	20	2	IV, 2	5787.54	80	8	III
5120.31	20	?	V?	5794.27	15	....	V
5134.73	40	?	IV?, 2	5804.06	15	....	V, 2
5160.34	60	?	IV?	5819.49*	80	—?	V?
5164.36	30	?	IV?, 6	5834.93	30	....	V
5180.32	60	8	IV	5838.69	60	3	IV, 4
5186.97	15	4	IV	5842.54	25	....	V, 2
5189.17	30	6	IV	5846.14	8	....	V, 6

TABLE I—*Continued*

λ	Intensities Arc	Intensities Fur.	Class No. Comps.	λ	Intensities Arc	Intensities Fur.	Class No. Comps.
5866.54......	100	?	IV?	6164.32......	10	....	V
5874.76......	30	....	V, 4	6213.09......	8	....	V
5877.81......	20	....	V	6221.92......	20	....	V
5893.50......	15	....	V, 2	6251.80......	15	....	V, 6
5900.64......	150	....	V, 6	6430.51......	100	....	V, 6
5903.89......	12	....	V, 6	6433.29......	30	....	V, 2
5928.21......	8	....	V, 6	6544.70......	60	....	V, 4
5934.21......	10	....	V, 6	6660.76*.....	150	....	V, 6
5957.77......	10	....	V	6677.36......	80	....	V, 6
5983.28......	100	10?	IV?	6701.25......	10	....	V
5986.17......	20	....	V	6723.66......	30	....	V, 2
5998.03*.....	20	....	V, 6	6739.95......	10	....	V, 2
6029.78......	12	....	V	6828.16......	20	....	V, 2
6031.88......	8	....	V	6876.51......	10	....	V
6045.54......	20	....	V	6908.16......	10	....	V
6148.11......	8	....	V, 2	6918.46......	10	....	V

NOTES TO TABLE

3175.80 Violet triplet resolved in arc

3271.98, 3272.07 } Measured by writer

3291.91, 3292.00 } Measured by writer

3299.62 *Cb* II line to violet

3319.25 Blend *Cb* II, *Cb* I

3326.61 *Cb* II line to violet

3349.51 Measured by writer

3369.21 *Cb* II line to red

3433.11 Faint line to red

3436.96 *Cb* II line to violet

3463.68, 3463.80 } Measured by writer

3465.85 Blend strong *Fe* line

3497.78 Blend strong *Fe* line

3510.26 May be blend with *Cb* I line given in Meggers' multiplet

3543.91, 3544.01 } Measured by writer

3554.52, 3554.65 } Measured by writer

3563.49, 3563.60 } Measured by writer

3637.84 *Cb* II line on red edge

3674.77 Probably pair of complex lines

3703.16 Slightly enhanced

NOTES TO TABLE—*Continued*

3704.15	Slightly enhanced
3709.43	Six-component *Cb* II line to violet
3713.79	*Cb* II line to violet
3716.19	Blend *Cb* II (violet), *Cb* I
3740.73 3740.84	Measured by writer
3766.15	Measured by writer
3770.89	*Cb* II line to violet
3796.46	Equal strength in arc and spark
3801.27	Measured by writer
3862.93	Measured by writer; *Cb* II line at λ 3863.05
3871.19	Measured by writer
3878.84 3878.95	Measured by writer
3883.14	Measured by writer
3922.32	Faint line to violet
3966.14 3966.29	Measured by writer; λ 3966.29 slightly enhanced
4311.33	Faint line to red
4749.60	Components partially resolved
4766.81	Blend *Cb* I, *Cb* II
4810.57	Components partially resolved
4848.27	Components partially resolved
5276.13	Components partially resolved
5671.06	Components partially resolved
5776.08	Measured by writer
5819.49	Very faint if present
5998.03	Very wide pattern
6660.76	Very wide pattern; probably more than six components

FEATURES OF THE *Cb* I SPECTRUM

Five hundred and seventy-eight of the 646 lines given in Table I belong to the neutral spectrum; 172 of these appear in the furnace spectrum with sufficient clearness to be assigned intensities; 33 others, very strong in the arc, are masked by foreign lines, usually belonging to carbon bands, and their presence is questioned in the furnace column. In general, the stronger neutral lines of the arc appear in the furnace, but an excitation considerably higher is evidently needed to bring out the complete neutral spectrum. The intensities of lines appearing at the low-temperature stage were estimated and used in the assignment of lines to temperature classes. These intensities were usually about one-third of those of the high-temperature stage, and to save space have been omitted from the table.

Certain outstanding groups appear, the lines of which are very strong in the arc, but show also in the furnace with considerable strength, and usually go into class III. Because of the absence of reversed lines and of the difficulty in producing the furnace spectrum, no lines are placed in the low-temperature classes I or II. Many lines obtained only at the highest temperature are placed in class IV. Among the numerous neutral lines in class V are some which are very strong in the arc but fail to appear in the furnace. A large proportion, however, being of moderate arc intensity, are scarcely to be expected in the furnace, by reason of the high excitation required for the columbium lines. This consideration rendered it useless to include the fainter lines of the arc spectrum.

A beginning in the analysis of the neutral lines was made in 1924 by Meggers,[1] who found three multiplets, and later Meggers and Kiess[2] gave two more, as well as several multiplets of *Cb* II. Almost all of the 56 lines of these neutral multiplets appear in the furnace spectrum, and half of them show at the lower furnace temperature. All are to the violet of λ 4218. In one of these multiplets are found the three ultimate lines of A. de Gramont[3]—λλ 4059, 4080, and 4101—which are the strongest lines, both of the arc and furnace spectra, in the region examined.

FEATURES OF THE *Cb* II SPECTRUM

The arc and spark spectra of columbium were photographed during this investigation as far as λ 2650. The ionized spectrum is very rich in the ultra-violet, many of its lines being strong enough in the arc above λ 3100 to be included in Table I. Some of these, though beyond the reach of the furnace, evidently are from low levels of the ionized atom, as they reverse widely in a strong spark. Many of the weaker *Cb* II lines are as yet unmeasured. To the red of λ 3500 the ionized lines become more scattered, and the spectrum above λ 4100 is chiefly made up of neutral lines. Some spark lines, usually of hazy structure, have not appeared in the arc spectrograms and may belong to the doubly ionized spectrum. Considerable variety

[1] *Journal of the Washington Academy of Sciences*, **14**, 442, 1924.

[2] *Journal of the Optical Society of America*, **12**, 417, 1926.

[3] *Comptes rendus*, **171**, 1106, 1920.

in the relative intensities in arc and spark appears among the ionized lines, and much may be learned as to the relative energy-levels of *Cb* II lines from a comparison of arc and spark spectra.

HYPERFINE STRUCTURE

A conspicuous feature of the columbium spectrum is that a large proportion of the lines are by no means sharp, but show, even under low excitation, the definite spreading which characterizes hyperfine structure. Only preliminary data on the extent and character of this phenomenon are now available, as a proper examination will require apparatus of high resolution. The figures after the class numbers in the table indicate, for lines clearly complex, the probable number of hyperfine components according to the present spectrograms. The components of the columbium patterns are so densely spaced, however, that these figures, for the most part 2, 4, or 6, are useful mainly as an indication of the relative widths of the patterns, since resolution, even of the wider-spaced components, was obtained for only a few lines. Of the 646 lines in Table I, 268 show clear evidence of hyperfine structure, and 57 of these have widths indicating at least six components in each case. For the few patterns showing partial resolution, the spacing of components is graduated, with the wider intervals on the violet side. These resemble the patterns of praseodymium,[1] except that some of the latter are graduated in the opposite direction.

Both neutral and ionized lines of columbium show hyperfine structure, and in neither group is its presence confined to lines requiring high excitation. An interesting instance of variation in structure among lines in the same multiplet is shown by the ultimate lines mentioned above—$\lambda\lambda$ 4059, 4080, 4101. The widths of their patterns, apparently of six, four, and two unresolved components, respectively, remain unchanged at the lowest furnace temperature.

Toward the red the average width of the columbium patterns increases. In the violet, complex lines have their components closely spaced, the total width of a six-component pattern being less than 0.2 A. In the green the greatest line-widths are about 0.33 A and

[1] King, *Mt. Wilson Contr.*, No. 368; *Astrophysical Journal*, **68**, 194, 1928; H. E. White, *Physical Review*, **34**, 1397, 1929.

comparable with those of praseodymium in this region; while in the red, widths near 0.5 A are found, although the first-order spectrograms show little resolution. The widest line observed in the spectrum is λ 6660.76, with a total width of about 0.75 A.

To resolve the columbium patterns will require an examination with the higher orders of the 75-foot spectrograph on Mount Wilson. Some preliminary spectrograms, made with a plane grating of insufficient size, gave partial resolution of a few of the wider patterns and showed the instrumental requirements for a detailed study. Several of these very wide lines showed in each case three components distinctly resolved, and these three made up less than half the total width of the line. Since the remainder of the pattern should consist of at least five closely spaced components, the wider lines of this element may be expected, if sufficient resolution is attained, to show eight or more components.

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A COMPARISON OF THE REVISED HARVARD PHOTOMETRIC AND THE INTERNATIONAL PHOTOVISUAL SYSTEMS BETWEEN THE SIXTH AND THE TENTH MAGNITUDES

By R. S. ZUG

ABSTRACT

The photovisual magnitudes of 297 stars have been determined with a probable error, ± 0.04 mag., on the International photovisual system, from photographs made at the Yerkes Observatory with the F. E. Ross 3-inch photovisual camera.

A comparison of the adopted photovisual with the corresponding photometric magnitudes from the revised Harvard photometry, *Harvard Annals*, **54**, 1908, yields the color-equation and scale-difference of the revised Harvard photometry (4-inch photometer only) with respect to the International photovisual system.

The color-equation, $Pv-Ptm$, was found for the spectral types B0, A0, F0, G0, K0, and M0 to be $+0.02$, 0.00, -0.04, -0.09, -0.19, and -0.32 mag., respectively. The *scale-differences*, $Pv-Ptm$, at the Harvard photometric magnitudes, 6.5, 7.5, 8.5, and 9.5, were found to be 0.00, $+0.12$, $+0.22$, and $+0.30$ mag.

The relation between the Harvard visual (in this article designated photometric) and the International[1] photovisual systems of magnitude was discussed in 1924 by Dr. Frederick H. Seares.[2] The standard photovisual magnitudes of the polar stars were compared directly with the corresponding Harvard visual magnitudes for these stars. In the present paper, the International photovisual magnitudes of 297 stars are first determined,[3] and then compared with the corresponding photometric magnitudes from the supplement to the revised Harvard photometry, *Annals of the Harvard College Observatory*, **54**. The present investigation covers only the interval from 6.5 to 10.0 mag. This interval is substantially that of the 4-inch meridian photometer of the Harvard College Observatory. The spectral types from the *Henry Draper Catalogue* are used as measures of color of the stars. The types of a few stars were taken from the *Harvard Bulletin*, No. 871.

[1] *Transactions of the International Astronomical Union*, **1**, 69, 1922.

[2] *Contributions from the Mount Wilson Observatory*, No. 288, 1925; *Astrophysical Journal*, **61**, 284, 1925.

[3] Seventy-five of the 297 photovisual magnitudes of the present list have been copied from F. E. Ross and R. S. Zug, "Magnitudes and Colors of the Eros Comparison Stars," *Astronomische Nachrichten*, **239**, 289, 1930. The remaining 222 result from a further utilization of the same photographs from which the magnitudes of *ibid.* were determined.

The 25 fields in which the magnitudes are investigated extend from 5 to 10.5 hours in right ascension, and from $+47^\circ$ to -17° in declination. Although the regions are not widely distributed over the sky, the investigation is believed to have yielded useful information concerning the relation of the Harvard photometric to the International photovisual system in regard to color-equation and scale-difference within the interval studied.

The 3-inch F. E. Ross photovisual camera of focal length 532 mm of the Yerkes Observatory was used in securing the photographs utilized in the present investigation. A filter of medium-yellow Jena glass, No. 4351, 3.5-mm thick, with plane surfaces accurately figured by Petitdidier, is placed before the objective. It transmits light of wave-length λ 4950 or longer, and cuts sharply at that point. The use of this filter with Cramer instantaneous isochromatic plates gives a spectral range from λ 4950 to λ 5900, which is practically identical with the region utilized at Mount Wilson ($\lambda\lambda$ 5000–5900) in determining the photovisual scale of the north polar sequence which was adopted by the International Astronomical Union as standard. Since a much larger difference in spectral sensitivity must exist to effect a noticeable change in color, we can assume the two systems to be the same in respect to color.[1]

DETERMINATION OF THE PHOTOVISUAL MAGNITUDES

Fifty photographs[2] of the 25 regions mentioned are available for the determination of the photovisual magnitudes. These photographs were made on nights when the transparency was exceptionally good, and presumably uniform over the sky. Each plate received a 20-minute exposure on the field in question, and two exposures of the same duration on the polar sequence, taken immediately before and after the exposure on the field. The exposures on the fields were made during such intervals that at each mid-exposure the field was at the altitude of the pole, thus practically eliminating errors due to atmospheric extinction. In the reduction of the magnitudes, dif-

[1] *Astrophysical Journal*, **61**, 285, 1925.

[2] These photographs were taken originally for the determination of the photovisual magnitudes of 636 Eros comparison stars of the list of A. Kopff. Similar photographs were taken simultaneously with the 3-inch F. E. Ross photographic camera for the determination of the photographic magnitudes.

ferential extinction was allowed for according to the tables of H. L. Vanderlinden.[1]

The two images of each of the polar stars and the single image of each field star were measured by comparison with a scale of graduated images. The scale used for the majority of the plates contained 23 images. The gradation in size of image was such that the interval between successive images corresponded to about 0.20 mag. Estimates were made to the fifth-of-a-scale division, or to 0.04 mag. After measurement of a plate, the mean scale-reading of each polar star was plotted against the corresponding International photovisual magnitude. The resulting smoothed curve was then entered with scale-readings of the field stars to obtain their magnitudes. The final

TABLE I

Star	*Pv* Mag.	Star	*Pv* Mag.	Star	*Pv* Mag.
2s	6.30	8	8.13	12	9.77
2r	6.32	4r	8.27	4s	9.83
3s	6.35	5r	8.63	7r	9.87
5	6.45	9	8.83	5s	10.06
6	7.06	10	9.06	13	10.37
7	7.55	6r	9.24	8r	10.46
3r	7.57	11	9.56	14	10.56

magnitude of a field star then resulted after correction for differential atmospheric extinction and for distance of the image from the center of the plate.

Theoretically, in the construction of the reduction-curve, scale-reading against photovisual magnitude of the polar stars, the magnitudes should first be corrected for distance of image from the center of the plate, and for differential atmospheric extinction. However, these corrections were found to be negligible for plates taken with the F. E. Ross camera, the sequence of stars all being comparatively near the pole, and they were disregarded. Table I gives a list of the polar stars used in forming the reduction-curves, with the corresponding photovisual magnitudes recommended by the International Astronomical Union.

The adopted photovisual magnitudes are given in Table II with other convenient information. Column 1 gives the current number

[1] *Annales de l'Observatoire de Belgique,* 3.

TABLE II

No.	B.D.	α 1900	δ 1900	Sp. Cl.	*Ptm* Mag.	*Pv* Mag.	*Pv*−*Ptm* Mag. (100)
1......	1115	$5^h18^m.0$	+45° 8′	A0	8.47	8.59	+12
2......	1270	19.9	43 55	K0	7.84	7.36	−48
3......	1272	20.4	43 17	A0	6.75	6.71	− 4
4......	1206	20.7	44 50	K0	7.72	7.76	+ 4
5......	1126	22.8	45 44	K0	8.20	8.29	+ 9
6......	1317	25.2	42 20	F0	7.88	8.19	+31
7......	1298	21.9	42 12	G5	6.76	6.65	−11
8......	1131	25.3	45 28	F8	7.82	7.96	+14
9......	1132	25.3	45 25	G5	7.92	8.08	+16
10......	1023	25.7	46 48	A0	7.54	7.84	+30
11......	1134	25.9	45 9	K0	8.42	8.53	+11
12......	1232	26.5	44 43	F5	7.82	7.80	− 2
13......	1137	26.8	45 11	B9	8.47	8.45	− 2
14......	1301	26.8	43 52	B9	7.18	7.24	+ 6
15......	1310	28.1	43 11	G5	7.34	7.33	− 1
16......	1143	28.2	45 10	F0	9.17	9.23	+ 6
17......	1247	28.9	44 15	F5	7.17	7.31	+14
18......	1252	30.2	44 32	G5	7.34	7.20	−14
19......	1150	31.7	45 23	G5	8.12	8.06	− 6
20......	1325	32.8	43 16	K5	7.14	7.02	−12
21......	*1376*	35.1	43 00	B3	6.99	7.16	+17
22......	1270	35.7	44 48	A2	7.67	7.59	− 8
23......	1338	37.0	43 31	G5	7.70	7.76	+ 6
24......	1052	40.8	46 40	K0	7.10	7.08	− 2
25......	1362	43.4	43 59	A2	7.45	7.62	+17
26......	1296	43.6	44 55	K0	8.67	8.72	+ 5
27......	1060	44.9	46 46	K0	7.68	7.74	+ 6
28......	1300	44.9	44 56	....	10.17	10.37	+20
29......	1310	47.8	44 58	....	9.72	9.11	−61
30......	1214	48.8	47 42	B9	7.59	7.77	+18
31......	1205	50.2	45 53	G5	6.56	6.65	+ 9
32......	1209	50.6	45 6	K0	8.17	8.13	− 4
33......	1216	52.0	45 37	A0	6.60	6.89	+29
34......	1075	52.1	46 31	B5	7.24	7.57	+33
35......	1224	53.6	45 10	F8	8.37	8.51	+14
36......	1225	53.9	45 10	A0	7.57	7.67	+10
37......	1230	55.6	45 2	F5	9.42	9.99	+57
38......	1235	57.1	45 34	A2	7.18	7.35	+17
39......	1091	57.2	46 33	B3	6.98	7.20	+22
40......	1353	57.2	44 16	F8	6.71	6.79	+ 8
41......	1247	6:00.8	45 4	K2	8.42	8.27	−15
42......	1248	0.9	45 33	A0	7.32	7.45	+13
43......	1105	2.8	46 46	K0	7.18	7.18	00
44......	1261	3.7	47 55	A2	6.84	6.80	− 4
45......	1381	3.9	44 58	G5	7.52	7.51	− 1
46......	1383	4.2	44 10	A2	7.52	7.51	− 1
47......	1257	4.5	45 3	B9	8.37	8.58	+21
48......	1259	4.9	45 5	K0	8.57	8.55	− 2
49......	1396	6.5	44 47	K0	7.72	7.64	− 8
50......	1399	7.4	44 37	A5	8.07	8.06	− 1
51......	1119	7.6	46 25	B8	7.28	7.58	+30
52......	1121	9.8	46 4	A5	7.44	7.66	+22
53......	1281	9.9	45 53	K2	7.67	7.66	− 1

TABLE II—*Continued*

No.	B.D.	α 1900	δ 1900	Sp. Cl.	*Ptm* Mag.	*Pv* Mag.	*Pv*−*Ptm* Mag. (100)
54......	1285	6ʰ11ᵐ6	+47° 25′	F5	7.00	6.92	− 8
55......	1288	14.0	45 9	Ma	8.77	8.63	−14
56......	1289	14.4	45 40	Ko	7.37	7.17	−20
57......	1135	15.0	46 13	A2	7.26	7.30	+ 4
58......	1441	18.1	44 54	....	9.47	9.55	+ 8
59......	1442	18.9	44 47	B9	7.07	8.11	+ 4
60......	1445	19.4	44 52	K5	8.92	8.91	− 1
61......	1387	19.9	48 51	F5	7.29	7.41	+12
62......	1456	21.8	44 57	B9	8.57	8.91	+34
63......	1310	25.4	47 1	K5	7.14	7.13	− 1
64......	1157	25.8	46 34	B8	6.78	7.05	+27
65......	1315	26.0	45 00	....	9.52	9.67	+15
66......	1490	28.9	44 48	F5	8.82	9.43	+61
67......	*1331*	31.7	44 59	K5	9.12	9.32	+20
68......	1508	32.8	44 57	A2	9.17	9.01	−16
69......	1509	33.0	44 25	G5	6.82	6.75	− 7
70......	1326	34.4	47 53	A2	7.54	7.58	+ 4
71......	1328	35.1	47 51	F5	7.27	7.36	+ 9
72......	1517	35.8	44 56	F8	9.32	9.51	+19
73......	1519	36.3	44 50	Ko	9.37	9.54	+17
74......	1525	37.5	44 37	G5	6.80	6.75	− 5
75......	1192	42.6	46 17	Ko	7.30	7.14	−16
76......	1197	44.3	46 38	Ko	7.56	7.46	−10
77......	1568	47.1	49 1	A3	6.87	6.95	+ 8
78......	1556	49.7	44 48	K5	8.22	8.01	−21
79......	1457	50.4	48 38	Ao	7.85	8.13	+28
80......	1460	52.1	48 45	Ko	7.45	7.49	+ 4
81......	1569	54.8	44 36	Ko	7.12	6.91	−21
82......	1386	56.4	47 25	F8	7.62	7.85	+23
83......	1388	56.6	47 13	F5	7.04	7.09	+ 5
84......	1383	56.8	45 13	Fo	7.62	7.65	+ 3
85......	1387	58.8	45 1	K5	8.17	7.83	−34
86......	1577	58.9	44 46	Ao	9.27	9.42	+15
87......	1584	7:00.9	44 11	G5	6.95	6.69	−26
88......	1394	3.6	45 25	Ko	7.77	7.69	− 8
89......	1411	5.8	47 48	A5	6.62	6.66	+ 4
90......	1413	6.1	47 27	F2	7.08	6.97	−11
91......	1404	7.5	45 4	....	9.47	9.57	+10
92......	1408	8.4	45 35	Ko	6.69	6.59	−10
93......	1410	8.6	45 24	A2	8.22	8.42	+20
94......	1420	9.0	47 49	A2	7.44	7.67	+23
95......	1415	10.8	45 18	F2	7.62	7.55	− 7
96......	1610	10.8	44 46	K2	9.17	9.18	+ 1
97......	1430	17.7	45 3	Ko	7.82	7.83	+ 1
98......	1270	20.9	46 43	A2	6.78	6.85	+ 7
99......	1535	21.0	48 17	G5	7.88	8.06	+18
100......	1272	21.0	46 29	F5	7.78	8.00	+22
101......	1537	21.4	48 8	Mb	6.90	6.76	−14
102......	1273	21.6	46 33	Ko	6.63	6.49	−14
103......	1633	23.1	49 5	A2	7.16	7.29	+13
104......	1282	26.9	46 23	K5	7.28	7.21	−7
105......	1548	27.3	48 25	A3	7.17	7.22	+ 5
106......	1549	27.7	48 55	Fo	6.91	6.96	+ 5

TABLE II—*Continued*

No.	B.D.	α 1900	δ 1900	Sp. Cl.	*Ptm* Mag.	*Pv* Mag.	*Pv−Ptm* Mag. (100)
107......	*1555*	$7^h30^m.9$	+47° 59′	Go	7.59	7.77	+18
108......	1296	33.7	46 51	Ko	7.91	8.14	+23
109......	1476	37.1	45 36	Ko	7.61	7.46	−15
110......	1569	37.7	48 38	Fo	7.60	7.72	+12
111......	1484	40.1	47 36	Ko	7.65	7.94	+29
112......	1673	40.9	49 37	F5	7.77	7.73	− 4
113......	1576	41.2	48 2	G5	8.08	8.16	+ 8
114......	1320	43.2	46 3	A2	7.06	7.12	+ 6
115......	1323	44.8	46 12	A3	6.53	6.56	+ 3
116......	1327	46.2	46 1	G5	7.57	7.48	− 9
117......	*1694*	51.6	48 53	Ko	7.42	7.45	+ 3
118......	*1510*	52.6	46 53	Go	7.84	8.00	+16
119......	1597	55.4	48 53	K5	7.58	7.29	−29
120......	*1348*	56.0	45 53	Ko	8.16	7.95	−21
121......	1530	8:00.0	47 7	A2	8.06	8.38	+32
122......	1612	1.0	48 28	K5	8.13	8.09	− 4
123......	1545	4.9	47 14	A2	7.69	7.88	+19
124......	1621	6.3	48 35	B9	6.75	6.89	+14
125......	1552	7.1	45 1	....	9.49	9.78	+29
126......	1553	9.1	47 6	K5	6.99	6.83	−16
127......	1560	11.5	47 20	G5	7.46	7.47	+ 1
128......	1568	15.7	45 40	Ko	8.10	8.01	− 9
129......	1576	17.3	45 17	Go	7.53	7.88	+35
130......	1639	17.9	48 5	F5	8.02	8.54	+52
131......	*1586*	20.9	44 58	....	9.72	10.00	+28
132......	1399	21.2	46 36	Ko	7.01	7.09	+ 8
133......	1410	23.6	46 37	Ko	8.24	8.46	+22
134......	1583	23.7	47 6	F5	7.34	7.55	+21
135......	1601	27.3	45 32	Fo	7.80	7.80	00
136......	1592	27.5	47 28	Ao	6.62	6.63	+ 1
137......	1765	30.9	44 48	....	9.67	9.69	+ 2
138......	1768	32.2	44 42	G	9.42	9.74	+32
139......	1769	32.5	44 35	Ko	8.37	8.39	+ 2
140......	1770	32.7	44 38	K2	8.22	8.35	+13
141......	1614	34.1	45 15	K7	8.92	8.66	−26
142......	1620	34.9	45 15	F2	7.87	7.90	+ 3
143......	1784	38.5	44 2	F5	7.22	7.19	− 3
144......	*1635*	39.9	44 55	K7	9.20	9.01	−19
145......	1642	43.1	45 3	Ao	7.72	7.99	+27
146......	1795	45.7	44 43	F2	9.02	9.62	+60
147......	1803	49.5	44 44	Ma	8.82	9.01	+19
148......	1458	49.8	46 9	Fo	6.93	6.89	− 4
149......	1807	50.5	44 49	G5	6.91	7.04	+13
150......	1809	52.2	44 49	Go	8.02	8.29	+27
151......	1810	52.4	44 42	F8	8.87	9.17	+30
152......	1463	52.7	46 9	Ao	6.61	6.70	+ 9
153......	1831	9:02.1	44 39	....	9.42	9.81	+39
154......	*1686*	3.3	44 53	F5	8.97	9.20	+23
155......	1688	5.1	45 13	Ko	7.52	7.25	−27
156......	1843	6.8	44 43	K5	8.32	8.27	− 5
157......	1848	9.2	44 40	Go	9.42	9.52	+10
158......	*1897*	9.4	42 51	Ko	7.82	7.71	−11
159......	*1700*	12.0	44 51	Mo	9.97	9.97	00

TABLE II—*Continued*

No.	B.D.	α 1900	δ 1900	Sp. Cl.	*Ptm* Mag.	*Pv* Mag.	*Pv−Ptm* Mag. (100)
160......	2197	$9^{\mathrm{h}}16^{\mathrm{m}}.2$	+40° 40′	K2	7.71	7.60	−11
161......	*1715*	17.7	44 53	Ko	9.27	9.17	−10
162......	1717	18.8	45 3	Ko	7.82	8.01	+19
163......	1861	18.8	44 27	Ko	8.02	8.10	+ 8
164......	1961	20.5	41 28	K2	7.72	7.49	−23
165......	1915	21.1	43 12	Fo	7.37	7.51	+14
166......	1963	21.4	41 38	F2	7.42	7.54	+12
167......	*1968*	23.1	40 53	Ko	8.50	8.48	− 2
168......	2016	23.7	42 42	F5	7.81	7.92	+11
169......	1943	32.9	43 36	Ko	6.63	6.58	− 5
170......	1944	33.1	43 24	Ko	8.37	8.53	+16
171......	2033	33.4	42 31	Ko	8.22	8.21	− 1
172......	2253	39.9	40 17	F2	7.17	7.26	+ 9
173......	2271	35.2	39 25	G5	6.96	7.04	+ 8
174......	2022	43.4	37 14	K5	6.92	6.64	−28
175......	2261	44.1	40 6	K5	6.76	6.74	− 2
176......	*2023*	45.1	36 58	F2	7.81	7.84	+ 3
177......	2266	45.7	40 5	A3	7.62	7.88	+26
178......	2001	44.3	36 21	Ko	7.29	7.16	−13
179......	2076	46.3	38 24	Fo	6.74	6.69	− 5
180......	2004	46.3	36 48	F5	7.72	7.76	+ 4
181......	2006	46.5	36 0	K5	7.30	6.92	−38
182......	2289	49.0	39 42	....	9.45	9.70	+25
183......	2086	51.9	35 35	K5	7.43	7.22	−21
184......	2096	57.9	38 33	F5	6.82	6.85	+ 3
185......	2101	59.8	35 9	Ko	8.02	8.05	+ 3
186......	2079	10:00.1	34 44	G5	8.37	8.36	− 1
187......	*2106*	1.0	34 51	G5	8.92	9.00	+ 8
188......	2088	2.6	34 0	G5	7.50	7.56	+ 6
189......	2089	2.9	34 44	Ko	7.62	7.69	+ 7
190......	*2113*	5.0	34 57	....	9.72	9.83	+11
191......	*2058*	5.1	36 58	Ko	7.40	7.25	−15
192......	*2116*	5.3	34 55	....	9.77	9.82	+ 5
193......	2122	10.3	35 41	A2	7.32	7.31	− 1
194......	*2123*	10.4	34 54	K7	9.32	9.30	− 2
195......	2119	11.9	31 24	Ma	7.88	7.81	− 7
196......	*1991*	13.0	29 52	G5	9.31	9.64	+33
197......	*1994*	14.1	29 51	Go	9.26	9.26	00
198......	2029	14.5	29 44	F8	9.11	9.13	+ 2
199......	2125	15.2	31 10	Ko	7.26	7.17	− 9
200......	2240	17.1	25 5	F5	7.81	7.89	+ 8
201......	2133	17.4	31 22	Ko	7.57	7.53	− 4
202......	2081	18.2	26 4	Ko	6.87	6.72	−15
203......	*2136*	19.4	30 52	G5	7.82	7.95	+13
204......	2249	20.7	25 14	Ko	7.31	7.27	− 4
205......	2014	21.4	30 11	F2	7.81	7.85	+ 4
206......	1897	23.1	27 26	Ko	8.06	7.94	−12
207......	2238	26.3	24 36	G5	7.86	8.07	+21
208......	2258	26.4	25 14	Ko	7.91	7.89	− 2
209......	2232	26.6	22 32	Fo	7.45	7.40	− 5
210......	*2260*	26.8	24 57	Fo	7.16	7.26	+10
211......	*2244*	28.3	23 52	A2	7.10	7.31	+21
212......	*2210*	28.5	20 49	F5	7.31	7.28	− 3

TABLE II—*Continued*

No.	B.D.	α 1900	δ 1900	Sp. Cl.	*Ptm* Mag.	*Pv* Mag.	*Pv*−*Ptm* Mag. (100)
213......	2236	10^h29^m0	+22° 7′	Ko	7.32	7.25	− 7
214......	*2240*	29.9	21 54	F2	7.65	7.81	+16
215......	2321	15.5	5 10	F8	8.16	8.03	−13
216......	*2352*	15.9	2 47	B3	6.66	6.43	−23
217......	*2324*	16.5	4 49	Ma	9.16	8.94	−22
218......	2328	16.9	− 0 15	G5	7.53	7.23	−30
219......	2344	17.0	+ 9 29	K5	6.97	6.53	−44
220......	*2431*	18.5	0 50	F	9.49	9.50	+ 1
221......	2332	18.4	− 0 24	G5	6.62	6.32	−30
222......	2649	19.0	+ 0 42	....	9.34	9.80	+46
223......	2217	19.1	11 6	Ko	6.84	6.78	− 6
224......	*2361*	19.3	2 54	Ko	6.71	6.54	−17
225......	2240	19.4	14 5	K	8.84	9.04	+20
226......	2337	19.7	− 0 18	Ao	7.88	7.93	+ 5
227......	*2484*	20.7	+19 54	A2	8.75	9.05	+30
228......	2365	20.8	3 26	Ao	6.75	6.78	+ 3
229......	2328	21.0	4 26	A2	6.63	6.63	oo
230......	2243	21.2	14 46	F8	8.79	8.99	+20
231......	2224	21.4	17 44	F8	7.42	7.36	− 6
232......	2244	21.4	14 8	Ko	7.41	7.38	− 3
233......	2341	21.6	− 0 29	Ko	6.78	6.29	−49
234......	*2211*	21.9	+11 49	G5	6.60	6.64	+ 4
235......	2122	22.1	16 17	K2	8.22	8.01	−21
236......	2330	22.1	4 33	F5	9.15	9.35	+20
237......	*2336*	22.3	4 49	Ko	9.36	9.41	+ 5
238......	2333	22.3	4 4	Ko	7.22	7.07	−15
239......	*2203*	22.5	14 47	F8	9.39	9.58	+29
240......	2123	23.1	16 17	F8	7.17	7.37	+20
241......	2231	23.3	17 38	G5	7.07	7.12	+ 5
242......	*2206*	23.4	14 52	Ko	7.14	6.83	−31
243......	2252	24.4	14 40	Go	8.79	8.88	+ 9
244......	2323	24.6	2 00	F5	6.85	6.82	− 3
245......	2325	25.3	2 40	Ko	7.12	6.93	−19
246......	2314	25.3	7 34	G5	7.37	7.30	− 7
247......	2379	26.4	3 22	Ko	6.58	6.51	− 7
248......	*2165*	26.9	9 56	Ko	8.12	8.11	− 1
249......	2347	27.1	5 10	Ko	7.21	7.07	−14
250......	2356	27.6	− 0 21	F8	8.88	8.95	+ 7
251......	2350	28.6	4 42	K2	9.16	9.12	− 4
252......	2351	29.1	4 38	K2	8.95	8.80	−15
253......	2269	31.4	14 34	F8	8.24	8.28	+ 4
254......	*2176*	32.0	9 54	Ko	8.52	8.47	− 5
255......	2280	31.7	13 23	Ao	7.52	7.85	+33
256......	2273	32.8	14 39	Ko	9.34	9.14	−20
257......	*2230*	33.0	14 47	G5	9.09	9.09	oo
258......	2144	33.6	16 39	F2	6.62	6.55	− 7
259......	*3016*	9:59.3	−15 10	Mo	9.11	8.77	−34
260......	3029	10:02.5	14 58	Fo	7.86	7.98	+12
261......	3039	4.3	14 59	G5	7.46	7.30	−16
262......	3098	4.5	12 52	Ma	7.34	6.99	−35
263......	*3044*	5.7	15 8	A2	8.91	9.21	+30
264......	*3046*	6.1	15 13	K2	7.11	6.91	−20
265......	2987	6.2	16 26	Ko	7.26	6.93	−33

TABLE II—*Continued*

No.	B.D.	α 1900	δ 1900	Sp. Cl.	*Ptm* Mag.	*Pv* Mag.	*Pv*−*Ptm* Mag. (100)
266......	3054	1c^{h}7^{m}6	−14° 34′	Ao	6.99	7.04	+ 5
267......	*3055*	7.8	15 13	F8	8.71	8.83	+12
268......	2985	8.3	7 57	G5	7.71	7.69	− 2
269......	2817	8.5	4 35	F2	7.50	7.43	− 7
270......	3109	8.7	6 53	F8	7.32	7.02	−30
271......	2819	8.8	4 43	A2	7.40	7.54	+14
272......	2989	9.2	7 30	Ko	7.06	6.88	−18
273......	2833	10.0	11 31	F5	7.18	7.19	+ 1
274......	2835	10.3	11 17	Ao	7.38	7.40	+ 2
275......	3028	11.3	5 20	B9	7.60	7.86	+26
276......	3030	11.7	10 13	....	9.56	9.80	+24
277......	3031	11.8	10 16	A5	8.61	8.72	+11
278......	3031	11.9	5 21	Ao	8.90	9.13	+23
279......	3129	11.9	12 36	G5	7.17	6.92	−25
280......	3037	14.1	10 40	Go	7.66	7.91	+25
281......	3039	14.3	10 14	F2	8.81	9.05	+24
282......	*2839*	14.4	5 8	Fo	9.00	9.26	+26
283......	2841	14.6	4 44	F2	7.56	7.80	+24
284......	*2379*	15.3	2 11	G5	9.17	9.24	+ 7
285......	2846	15.7	4 52	A2	6.96	6.95	− 1
286......	3043	15.7	5 41	A5	7.35	7.28	− 7
287......	2847	16.1	4 55	K2	6.96	6.67	−29
288......	2906	17.7	8 54	Ma	7.22	7.20	− 2
289......	*3132*	18.5	3 9	K5	6.67	6.42	−25
290......	2861	19.1	4 26	Ko	7.36	7.37	+ 1
291......	*2383*	19.5	2 7	K2	8.87	8.95	+ 8
292......	3030	20.9	7 21	K2	7.96	7.77	−19
293......	3062	21.3	5 56	Ao	6.91	6.98	+ 7
294......	2921	21.7	3 53	Ko	6.60	6.59	− 1
295......	*2873*	22.0	5 10	F5	8.85	9.17	+32
296......	*2391*	22.9	2 13	F2	8.27	8.21	− 6
297......	*2403*	27.0	2 8	Ko	8.17	8.02	−15

assigned the star; column 2, the B.D. number according to the notation of the *Henry Draper Catalogue.* Columns 3 and 4 contain respectively the right ascension and declination for 1900. Column 5 gives the spectral type (where known) from the *Henry Draper Catalogue* (or in the case of a few stars, from *Harvard Bulletin*, No. 871); column 6, the photometric magnitude from the revised Harvard photometry, *HA*, **54**; column 7, the photovisual magnitude as determined in the present investigation; and column 8, the difference between these two, $PV-Ptm$.

In most cases, the adopted photovisual magnitude is the mean of four determinations from different plates. In the case of a few stars, one image was obscured or defective and only three determinations

were possible. For the stars in the first or last field, and on the side opposite the second or next to last field, only two determinations were possible since no overlapping with an adjacent field occurs.

The probable error of a single photovisual magnitude from one plate was found from the residuals to be ± 0.076 mag.; from four plates, ± 0.04 mag.

REDUCTION OF THE REVISED HARVARD PHOTOMETRIC TO THE INTERNATIONAL PHOTOVISUAL SYSTEM

In the reduction of one system of magnitude to another, the relative color-equation, scale-difference, and zero-point of the two must be determined. In the present paper, the scale-difference is found by direct comparison at different magnitudes. Therefore, zero-point is automatically considered, and needs no separate discussion. An approximate color-equation can be obtained if we may assume that the stars of different spectral types occur with the same frequency in the different magnitudes, and that the scale-corrections vary linearly according to magnitude. Such a color-equation will include the average scale-correction over the interval investigated. Summing the differences $Pv - Ptm$, of Table II, column 8, according to spectral type, and computing the mean difference for the stars of each class, we get the approximate color-equation. The result is presented in Table III. Column 1 contains the spectral class; column 2, the number of observations; column 3, the mean difference for each spectral class; and column 4, the smoothed values from column 3. The smoothed values in column 4 constitute the approximate color-equation.

The photometric magnitudes in Table II were next provisionally corrected for color with the use of the results in Table III, and new differences, $Pv - Ptm$ found. These were then summed in five groups according to magnitude (Pv), and the mean difference found for each group. The scale-corrections in Table IV result from this operation. In Table IV, column 1 contains the number of observations in each group; column 2 contains the mean magnitudes of the groups; column 3, the mean differences, $Pv - Ptm$; and column 5, the smoothed values for the different magnitudes. The values in column 5 were found to remain unchanged in a redetermination after the application of a new color-equation, described in the following paragraph.

After correction of the photometric magnitudes for scale-difference according to Table IV, a new color-equation differing slightly from the first provisional one was found. The mean values of the

TABLE III

Sp. Cl.	No. Obs.	*Pv−Ptm*	Col. Eq.	*Pv−Ptm* Revsd.	Col. Eq. Revsd.
B3	3	+5	+13	+9	+16
B5	1	32	13	30	15
B8	2	28	12	27	14
B9	7	17	12	16	14
A0	20	13	12	15	14
A2	22	10	12	10	13
A3	4	10	12	17	13
A5	6	6	12	7	12
F0	13	8	11	7	10
F2	13	4	11	9	10
F5	23	15	10	12	8
F8	16	9	9	5	6
G0	10	15	7	9	5
G5	31	+1	+2	+1	+1
K0	71	−4	−4	−5	−5
K2	12	11	7	15	7
K5	17	14	12	11	11
K7	3	13	14	29	14
M	8	−9	−20	−14	−18

TABLE IV

Observed			Smoothed	
No.	Mean Mag.	Scale Corr.	*Pv* Mag.	Scale Corr.
			6.5	−14
56	6.73	−12		
			7.0	−8
57	7.25	−6		
			7.5	−2
62	7.73	+1		
			8.0	+3
59	8.37	+7		
			8.5	+8
			9.0	+12
32	9.30	+15		
			9.5	+16
			10.0	+20

quantities, *Pv−Ptm*, for each spectral class are given in column 5 of Table III, while column 6 gives the smoothed values.

It might be expected that a dependence of color-equation on magnitude would exist. An examination of the data for each of the five groups of magnitude used in determining the scale-correction re-

vealed that no reliable determination of the dependence was possible, the number of observations being insufficient. The trend of the result indicated an increase in color of not more than 0.02 $(m-m_0)$. This result was deemed not sufficiently definite to incorporate in the color-equation, and a mean color-equation was adopted.

According to the conventional scheme of color-equation depending on color-index, the equation becomes zero for A0 stars, the color-index being zero. We shall assign a color-equation of zero to the stars of spectral class A0. The amount of the color-equation for stars of this type as derived, +0.14 mag., may be subtracted from the color-

TABLE V

Sp. Cl.	$Pv-Ptm$ (Mag.)	$Y-Mt.W$
B0	+0.02	−0.02
B5	+ .01	− .01
A0	.00	.00
A5	− .02	+ .01
F0	.04	.01
F5	.06	.03
G0	.09	.03
G5	.13	.03
K0	.19	.02
K5	.25	.01
M0	−0.32	+0.06

TABLE VI

Ptm Mag.	$Pv-Ptm$ (Mag.)	$Y-Mt.W$
6.5	0.00	−0.02
7.0	+ .06	.00
7.5	.12	+ .02
8.0	.17	.03
8.5	.22	.03
9.0	.26	.02
9.5	.30	.02
10.0	+0.34	+0.09

equation of Table III, column 6, and added to the scale-corrections of Table IV, column 5. The resulting color-equation will be zero for stars of spectral class A0, and the scale-corrections will be the true ones.

The adopted color-equation and scale-corrections are given in Tables V and VI, columns 1 and 2, and are illustrated in Figures 1 and 2. The adopted scale-corrections are given with the argument, photometric magnitude.

COMPARISON WITH RESULTS OF MOUNT WILSON

In his reduction of the Harvard visual scale to the International (Mount Wilson) photovisual system, Seares finds for the bright stars a constant correction of −0.08 mag. and a color-equation of $-0.09C$ at a mean magnitude of about 3. C is the color-index on the International system. From the fifth to the tenth magnitude he finds a linear color-equation of from $-0.11C$ to $-0.17C$. By adopting $-0.14C$ as a mean value, and by using the color-indices of stars of

different spectral types as determined by Seares,[1] we may compute his color-corrections according to spectral type.

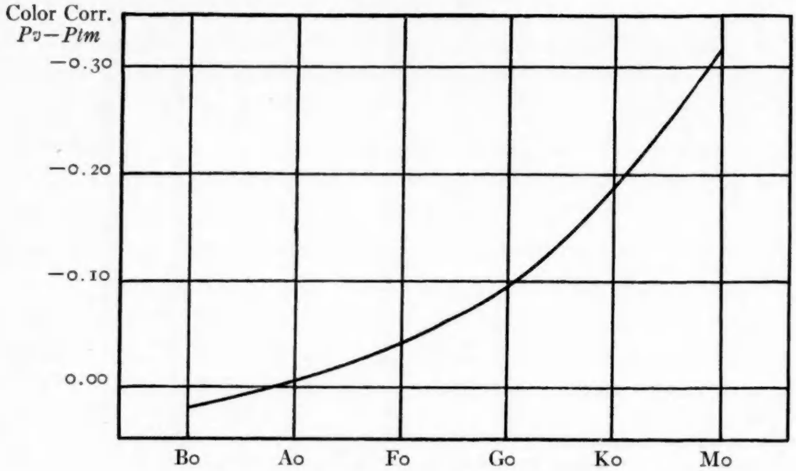

FIG. 1.—Adopted color-equation, $Pv-Ptm$

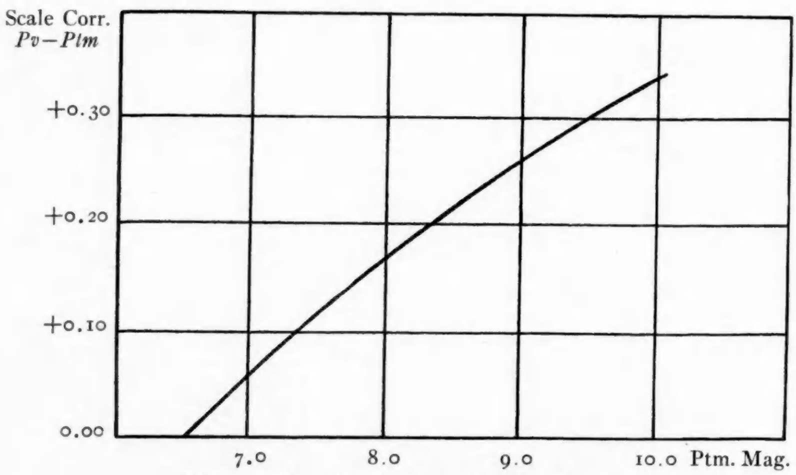

FIG. 2.—Adopted scale-correction, $Pv-Ptm$

Column 3 of Table V gives the values $Y-Mt.W$, found by subtracting from the color-correction determined in the present investigation the corresponding correction derived as outlined above from the results of Seares.

[1] *Mt. Wilson Contr.*, No. 226; *Astrophysical Journal*, **55**, 198, 1922.

Column 3 of Table VI contains the differences of scale-correction, $Y - Mt.W$, as determined in the present investigation and by Seares, for points in common. It will be noticed that exact agreement is reached for stars of revised Harvard magnitude 7.0. All photometric magnitudes of 7 or fainter which were utilized by Seares were obtained with the 4-inch meridian photometer of the Harvard College Observatory (for the very faint stars the 12-in. photometer was used). Magnitudes brighter than 7 were obtained with the 2-inch photometer. It is possible that the joining of the curves of scale-correction of the 2- and 4-inch photometers is responsible for the slightly different scale-corrections found by the two methods for magnitudes from 6.5 to 8.0. In the present investigation, it seemed advisable not to attempt to join the curve of scale-correction of the 4-inch to that of the 2-inch, but to consider the scale of the 4-inch photometer exclusively. In the case of the scale-correction for the Harvard magnitude 10.0, the different scale-correction obtained by Seares is apparently due to the smoothing of the curves for the 4- and 12-inch photometers. The value obtained in the present investigation is merely an extrapolation from the correction determined for 9.5 mag., and could apply only to magnitudes obtained with the 4-inch photometer.

An application of the adopted color- and scale-corrections to the Harvard magnitudes used reduces the average difference, $Pv - Ptm$, from 0.14 to 0.09 mag. The precision of the corrections is believed to be considerably within this limit. The adopted photovisual magnitudes have a probable error of ± 0.04 mag. It seems likely that most of the corrected average residual of 0.09 mag. must be attributed to the scattering of color-index for spectral type, if we can assume the substantial accuracy of the revised Harvard photometry and the spectral classification of the *Henry Draper Catalogue*. For the purpose of reduction of the revised Harvard magnitudes to the International system it undoubtedly would be more desirable to have as a measure of the color of the stars the color-indices. However, since in general these are unknown at present, the only practical method must employ a color-correction depending on spectral type.

YERKES OBSERVATORY
September 1930

THE DISTRIBUTION OF ABSOLUTE MAGNITUDES AMONG K STARS BRIGHTER THAN THE SIXTH APPARENT MAGNITUDE AS DETERMINED FROM PARALLACTIC AND PECULIAR VELOCITIES[1]

By GUSTAF STRÖMBERG

ABSTRACT

1. The distribution of absolute magnitudes among stars of spectral types K0–K2 and K3–K9 and brighter than apparent magnitude 6.0 has been determined from the distributions of parallactic and peculiar reduced proper motions and of radial velocities by methods described in *Mount Wilson Contributions* Nos. 395 and 410 (Table V). The number of proper-motion stars in the two groups is 1058 and 375, respectively; of radial-velocity stars, 849 and 348.

2. The distribution of absolute magnitudes for spectral types K0–K2 shows four distinct maxima: *bright giants* at −2.5, *normal giants* at +0.3, *faint giants* at +2.7, and *dwarfs* at +6.1. The relative proportion of stars in the four groups is 13.9, 78.3, 6.5, and 1.3 per cent.

For the interval K3–K9 the distribution shows three distinct maxima: *supergiants* at −4.5, *normal giants* at −0.1, and *dwarfs* at +6.7, the relative proportions of stars being 7.1, 90.5, and 2.4 per cent, respectively.

3. The distributions of reduced angular parallactic and peculiar motions and of the corresponding linear motions are given in Tables II–IV. Values are also given for the computation of the mean absolute magnitude for a group of stars from the mean apparent magnitude and the mean of the logarithms of the τ- or υ-components.

The methods for determining the distribution of absolute magnitudes from peculiar and parallactic motions outlined in *Mount Wilson Contributions* Nos. 395[2] and 410[3] have been applied to stars of spectral type K brighter than the sixth apparent magnitude, with the results given in this paper.

All stars in the *Draper Catalogue* classified as K and brighter than the sixth apparent magnitude were included. To these were added stars classified as K by the Mount Wilson observers, but not given as K in the *HD*, where they appear mostly as G5 or Ma stars. The stars were divided into two groups including spectral types K0–K2 and K3–K9, respectively.

The apparent magnitudes were taken from the *HD*. In the case of doubles the apparent magnitude of the brighter star was used as given in the *HD*, or was derived from the difference in brightness

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington,* No. 418.

[2] *Astrophysical Journal,* **71,** 163, 1930.

[3] *Ibid.,* **72,** 111, 1930.

between the components. Certain double stars whose combined apparent magnitude is brighter than six, but none of whose components attains this brightness, were thus excluded.

The proper motions have been taken from Boss's *Catalogue*. Raymond's corrections have been applied to the proper motions in declination.

The total number of stars in the two groups is 1058 and 375, respectively. Since certain stars are classed differently in the *HD* and by the Mount Wilson observers, the groups overlap each other partly and also, to a small extent, the group of M stars studied in *Contribution* No. 411.[1] Thus, a star classified as K5 in the *HD* and as K2 at Mount Wilson appears in both groups of K stars. It should also be noted that a very large group of stars is classified uniformly as K0 in the *HD*, whereas a great number of these stars are classified at Mount Wilson as G8 or G9. The present results for the early K stars are thus to a large extent representative for stars of types G8–K2 on the Mount Wilson scale.

From the two spectral groups were first subtracted, respectively, 14 and 9 stars having very large reduced proper motions. All of these stars have well-determined trigonometric parallaxes, and their absolute magnitudes are thus known individually with fair accuracy. Further, 153 and 63 stars whose angular distances from the apex or antapex are less than 30° were omitted from the two groups in analyzing the parallactic motions.

Among the K stars brighter than the seventh magnitude are 849 and 348 stars, respectively, for which spectrographically determined absolute magnitudes M_s and radial velocities V are available. Many of these are unpublished and were kindly put at my disposal by Dr. Adams. The group motions for these stars, computed for groups arranged according to absolute magnitude, are given in Table I.

The distribution of the peculiar radial velocities V' was derived for each absolute-magnitude group separately and combined with the distribution of sin λ. The distribution of log $|V_0+(\epsilon/\sin\lambda)|$ was then determined and smoothed graphically. This computation was based on the assumption that the distribution of the linear peculiar tangential velocities ϵ is identical with that of the peculiar radial

[1] *Astrophysical Journal*, 72, 117, 1930.

velocities V' and that there is no correlation between $|\epsilon|$ and $\sin\lambda$. Similarly, the distribution of $\log|V'|=\log|\epsilon|$ was determined for each group separately. The distributions of $y_1=\log|V'|-1.6756$ and $y_2=\log|V_0+(\epsilon/\sin\lambda)|-1.6756$, reduced to a total of 1000 stars, are shown in Tables II and III. The values of the absolute magnitudes at which the transition from one distribution to the next is supposed to occur are given at the head of the table between the columns. The distribution for the dwarfs, being based on only a few stars, is very uncertain and has not been used for determining the absolute magnitudes. The mean values of y_1 and y_2 for the different groups of

TABLE I

Ko–K2			K3–K9		
M_S	No.	V_0	M_S	No.	V_0
		km/sec.			km/sec.
<-1.0	27	19.4	<-1.0	40	22.7
-0.9 to 0.0	268	22.3	-0.9 to -0.5 ..	54	13.6
$+0.1$ to $+0.8$	474	21.2	-0.4 to -0.1 ...	99	19.6
$+0.9$ to $+2.6$	52	21.0	0.0 to $+0.3$ ...	93	26.7
			$+0.4$ to $+1.0$ ...	51	35.1

absolute magnitude are given at the bottom of Tables II and III. These numbers are analogous to the values given for the M stars in *Contribution* No. 411,[1] and may be used to derive mean absolute magnitudes by means of the formulae

$$\overline{M}=5\,\overline{\log|\tau|}+\overline{m}-5\,\bar{y}_1$$
$$\overline{M}=5\,\overline{\log|v|}+\overline{m}-5\,\overline{\log\sin\lambda}-5\,\bar{y}_2\,.$$

The second formula can be used only when the stars are well distributed over the sky.

The distributions of $x_1=\log|\tau|+0.2m$ and $x_2=\log|v|+0.2m-\log\sin\lambda$ are given in Table IV. For values of x_1 and x_2 less than -1.7 the frequencies are determined by the methods outlined in *Contributions* Nos. 395 and 410; otherwise no smoothing has been done. The observed and computed distributions are given under the headings "O" and "C," respectively.

[1] *Ibid.*

TABLE II

Ko–K2

y	$F(y_1)dy$					$F(y_2)dy$				
	−1.0	0.0	+2.0	+4.5		−1.0	0.0	+2.0	+4.5	
−3.6	1	0	0	0	1	0	0	0	0	0
3.5	0	1	0	0	0	0	0	0	0	0
3.4	0	0	1	1	0	1	0	0	0	0
3.3	1	0	0	0	0	0	0	1	1	0
3.2	0	0	0	0	1	0	1	0	0	1
3.1	1	1	0	0	0	0	0	0	0	0
3.0	1	0	1	1	1	0	0	0	0	0
2.9	1	1	1	0	1	0	0	0	0	0
2.8	1	1	1	1	1	1	0	1	1	0
2.7	2	1	1	1	1	1	1	0	0	1
2.6	2	1	1	1	2	1	0	1	1	0
2.5	3	2	1	1	2	1	1	1	1	1
2.4	4	2	2	1	3	1	1	1	1	1
2.3	5	3	2	2	3	2	1	1	1	1
2.2	6	4	3	3	4	2	1	1	1	1
2.1	7	4	3	3	5	3	1	2	2	1
2.0	9	5	5	4	7	4	2	2	2	2
1.9	10	7	6	5	8	5	2	3	3	2
1.8	14	9	8	6	11	6	3	4	4	3
1.7	18	11	9	8	13	7	4	5	5	4
1.6	21	14	12	10	17	9	5	6	6	5
1.5	24	17	15	12	22	11	6	7	7	6
1.4	27	20	18	15	25	14	7	9	9	8
1.3	31	24	22	19	30	17	9	12	12	10
1.2	36	29	27	23	36	20	13	15	15	13
1.1	41	35	39	29	43	24	17	18	19	17
1.0	48	46	51	37	51	27	23	21	22	25
0.9	55	59	62	46	63	32	32	26	28	38
0.8	66	74	70	59	75	40	41	34	35	58
0.7	90	85	78	82	82	49	52	45	44	82
0.6	122	97	85	100	85	70	72	62	58	101
0.5	134	101	91	102	85	106	91	84	76	113
0.4	134	97	93	101	84	133	115	107	97	114
0.3	36	86	91	94	82	134	120	120	111	112
0.2	22	70	80	77	72	114	116	120	114	95
−0.1	13	46	57	60	55	76	98	105	108	77
0.0	8	27	38	42	27	40	80	84	86	54
+0.1	4	14	22	32	2	22	48	56	64	36
0.2	2	5	4	18	0	14	22	29	36	16
0.3	0	1	0	4	0	8	10	12	20	2
0.4	0	0	0	0	0	4	4	4	8	0
0.5	0	0	0	0	0	1	1	1	2	0
+0.6										
$\bar{y}$..	−0.787	−0.647	−0.611	−0.544	−0.704	−0.476	−0.371	−0.362	−0.346	−0.441

TABLE III

K3–K9

y	$F(y_1)dy$						$F(y_2)dy$					
	−1.0	−0.4	0.0	+0.4	+4		−1.0	−0.4	0.0	+0.4	+4	
4.0	1	0	0	0	0	0	0	0	0	0	0	0
3.9	0	0	0	0	0	0	0	0	0	0	0	0
3.8	0	0	0	0	0	0	0	0	0	0	0	0
3.7	0	1	0	0	0	0	0	0	0	0	0	0
3.6	1	0	1	0	0	0	0	0	0	0	0	0
3.5	0	0	0	1	1	0	0	0	0	0	0	0
3.4	0	0	0	0	0	0	0	1	0	0	0	0
3.3	1	0	0	0	0	1	1	0	0	0	0	0
3.2	1	1	0	0	0	0	0	0	1	0	1	0
3.1	1	0	1	0	1	0	0	0	0	1	0	0
3.0	1	1	0	1	0	0	0	0	0	0	0	0
2.9	2	1	0	0	0	0	0	0	0	0	0	0
2.8	2	1	1	0	1	1	1	1	1	0	1	0
2.7	2	1	1	1	1	0	0	0	0	1	0	1
2.6	3	1	1	1	1	0	1	1	1	0	0	0
2.5	4	2	1	1	1	1	1	1	1	0	1	0
2.4	5	2	2	1	2	1	1	1	1	1	1	1
2.3	6	3	2	2	2	1	1	1	1	1	1	0
2.2	8	3	2	2	2	1	1	1	1	1	1	0
2.1	10	4	3	2	3	2	1	1	2	1	1	1
2.0	12	6	4	3	4	2	2	2	2	2	2	1
1.9	15	7	5	4	5	2	2	2	3	2	2	1
1.8	19	9	6	5	6	3	3	3	4	3	3	1
1.7	22	11	8	6	8	4	4	4	5	3	4	2
1.6	27	14	10	8	10	5	5	5	6	4	5	2
1.5	34	17	12	11	12	6	6	6	7	5	6	3
1.4	40	22	15	16	14	8	7	8	9	6	7	3

−1.3	47	27	19	25	16	10	8	10	12	8	9	4
1.2	54	35	23	35	20	12	10	16	15	10	12	5
1.1	60	50	31	44	25	15	12	25	21	13	15	6
1.0	67	64	36	51	33	19	14	41	27	17	22	8
0.9	73	77	42	59	42	23	18	58	35	22	29	10
0.8	77	88	56	65	56	31	23	66	43	35	35	13
0.7	77	95	70	71	69	36	31	82	53	64	43	17
0.6	73	98	88	80	82	42	54	91	66	86	55	22
0.5	67	97	112	83	92	56	92	99	80	96	70	35
0.4	59	85	120	89	97	70	185	103	92	100	89	64
0.3	44	70	120	92	98	88	227	106	103	101	98	86
0.2	32	55	105	91	93	112	124	103	112	101	102	96
−0.1	23	33	69	78	80	120	70	80	116	100	103	100
0.0	15	17	25	45	56	120	48	46	92	97	101	101
+0.1	9	2	8	18	36	105	26	26	50	70	80	101
0.2	4	0	1	7	17	69	13	8	26	34	52	100
0.3	2	0	0	2	8	25	6	2	10	10	28	97
0.4	0	0	0	0	4	8	2	0	2	4	13	70
0.5	0	0	0	0	2	1	0	0	0	1	6	34
0.6	0	0	0	0	0	0	0	0	0	0	2	10
0.7	0	0	0	0	0	0	0	0	0	0	0	4
0.8	0	0	0	0	0	0	0	0	0	0	0	1
+0.9	0	0	0	0	0	0	0	0	0	0	0	
$\bar{y}$..	−0.904	−0.720	−0.558	−0.568	−0.497	−0.258	−0.367	−0.482	−0.384	−0.333	−0.291	−0.033

TABLE IV

x	K0–K2						K3–K9					
	$F(x_1)dx$			$F(x_2)dx$			$F(x_1)dx$			$F(x_2)dx$		
	O	C	O−C	O	C	O−C	O	C	O−C	O	C	O−C
−4.3	....	....	....	....	....	....	0	0.1	− 0.1	....	....	....
4.2	....	....	....	....	....	....	0	0.1	− 0.1	....	....	....
4.1	....	....	....	....	....	....	0	0.1	− 0.1	....	....	....
4.0	0	0.1	− 0.1	....	....	....	0	0.1	− 0.1	....	....	....
3.9	0	0.1	− 0.1	....	....	....	0	0.2	− 0.2	....	....	....
3.8	1	0.1	+ 0.9	....	....	....	1	0.3	+ 0.7	0	0.0	0.0
3.7	0	0.1	− 0.1	....	....	....	0	0.3	− 0.3	1	0.1	+ 0.9
3.6	0	0.2	− 0.2	....	....	....	0	0.4	− 0.4	0	0.1	− 0.1
3.5	0	0.2	− 0.2	0	0.1	− 0.1	0	0.5	− 0.5	0	0.1	− 0.1
3.4	0	0.3	− 0.3	1	0.1	+ 0.9	1	0.7	+ 0.3	0	0.2	− 0.2
3.3	1	0.4	+ 0.6	0	0.2	− 0.2	0	0.8	− 0.8	0	0.2	− 0.2
3.2	0	0.5	− 0.5	0	0.3	− 0.3	0	1.0	− 1.0	1	0.2	+ 0.8
3.1	1	0.6	+ 0.4	0	0.3	− 0.3	1	1.3	− 0.3	0	0.4	− 0.4
3.0	1	0.8	+ 0.2	1	0.4	+ 0.6	1	1.5	− 0.5	1	0.4	+ 0.6
2.9	1	1.3	− 0.3	0	0.4	− 0.4	1	1.9	− 0.9	1	0.5	+ 0.5
2.8	1	1.5	− 0.5	0	0.5	− 0.5	1	2.3	− 1.3	1	0.6	+ 0.4
2.7	1	1.7	− 0.7	1	0.7	+ 0.3	2	2.9	− 0.9	1	0.8	+ 0.2
2.6	2	2.0	0.0	1	0.9	+ 0.1	2	3.7	− 1.7	1	0.9	+ 0.1
2.5	2	2.4	− 0.4	1	1.3	− 0.3	2	4.6	− 2.6	2	1.0	+ 1.0
2.4	3	3.0	0.0	1	1.5	− 0.5	3	5.6	− 2.6	2	1.2	+ 0.8
2.3	4	4.0	0.0	1	1.7	− 0.7	4	6.8	− 2.8	3	1.5	+ 1.5
2.2	5	4.8	+ 0.2	2	2.0	0.0	5	8.3	− 3.3	4	1.9	+ 2.1
2.1	6	5.8	+ 0.2	3	2.5	+ 0.5	6	10.0	− 4.0	5	2.4	+ 2.6
2.0	7	6.9	+ 0.1	3	3.3	− 0.3	8	11.9	− 3.9	6	3.0	+ 3.0
1.9	9	8.8	+ 0.2	4	4.0	0.0	10	14.1	− 4.1	7	3.9	+ 3.1
1.8	12	10.6	+ 1.4	5	5.1	− 0.1	12	16.7	− 4.7	9	4.9	+ 4.1
1.7	9	12.8	− 3.8	2	6.4	− 4.4	24	19.4	+ 4.6	3	6.1	− 3.1
1.6	18	15.3	+ 2.7	11	7.8	+ 3.2	45	22.2	+22.8	16	7.9	+ 8.1
−1.5	15	18.7	− 3.7	8	9.3	− 1.3	25	25.4	− 0.4	6	11.2	− 5.2

−1.4	24	22.9	+1.1	16	11.4	+4.6	36	28.9	+7.1	27	15.8	+11.2
1.3	34	27.9	+6.1	13	14.8	−1.8	53	33.4	+19.6	27	21.3	+5.7
1.2	27	33.8	−6.8	17	19.6	−2.6	35	39.2	−4.2	21	26.6	−5.6
1.1	38	41.0	−3.0	24	25.0	−1.0	40	44.6	−4.6	19	32.2	−13.2
1.0	46	49.4	−3.4	28	31.0	−3.0	57	49.0	+8.0	40	38.4	+1.6
0.9	52	57.5	−5.5	35	36.7	−1.7	75	53.6	+21.4	31	46.6	−15.6
0.8	60	62.4	−2.4	55	42.0	+13.0	71	57.9	+13.1	74	58.7	+15.3
0.7	69	66.8	+2.2	57	48.5	+8.5	55	62.2	−7.2	59	72.5	−13.5
0.6	74	70.3	+3.7	51	56.6	−5.6	56	66.5	−10.5	67	82.2	−15.2
0.5	80	73.1	+6.9	71	67.8	+3.2	53	68.3	−15.3	74	80.2	−6.2
0.4	74	74.3	−0.3	72	80.2	−8.2	69	67.5	+1.5	87	76.4	+10.6
0.3	73	72.5	+0.5	89	89.9	−0.9	69	64.0	+5.0	61	75.2	−14.2
0.2	70	66.9	+3.1	93	92.9	+0.1	59	55.5	+3.5	72	71.0	+1.0
−0.1	57	56.2	+0.8	82	88.2	−6.2	29	43.2	−14.2	63	63.0	0.0
0.0	41	42.2	−1.2	63	75.5	−12.5	35	30.0	+5.0	45	52.6	−7.6
+0.1	29	29.9	−0.9	51	58.4	−7.4	16	20.1	−4.1	61	40.7	+20.3
0.2	15	18.8	−3.8	55	40.5	+14.5	8	13.3	−5.3	32	29.2	+2.8
0.3	13	9.6	+3.4	36	25.5	+10.5	3	8.5	−5.5	13	18.9	−5.9
0.4	9	6.0	+3.0	15	15.6	−0.6	5	5.2	−0.2	16	11.9	+4.1
0.5	5	4.3	+0.7	9	9.8	−0.8	0	3.2	−3.2	10	7.3	+2.7
0.6	1	3.3	−2.3	9	6.7	+2.3	3	2.2	+0.8	3	4.3	−1.3
0.7	2	2.5	−0.5	3	4.6	−1.6	3	1.6	+1.4	0	2.5	−2.5
0.8	2	1.8	+0.2	1	3.3	−2.3	2	1.5	+0.5	0	1.7	−1.7
0.9	1	1.3	−0.3	3	2.3	+0.7	3	1.7	+1.3	3	1.7	+1.3
1.0	1	1.0	0.0	3	1.6	+1.4	0	2.1	−2.1	3	1.9	+1.1
1.1	2	0.7	+1.3	0	1.1	−1.1	3	2.5	+0.5	0	2.2	−2.2
1.2	0	0.4	−0.4	2	0.8	+1.2	0	2.7	−2.7	10	2.3	+7.7
1.3	0	0.1	−0.1	0	0.5	−0.5	0	2.7	−2.7	0	2.4	−2.4
1.4	1	0.0	+1.0	0	0.3	−0.3	3	2.4	+0.6	0	2.4	−2.4
1.5	1	0.0	+1.0	2	0.1	+1.9	3	1.8	+1.2	6	2.3	+3.7
1.6	0	0.0	0.0	0	0.0	0.0	2	1.0	+1.0	3	2.2	+0.8
1.7	....	....	....	....	....	....	0	0.4	−0.4	0	1.7	−1.7
1.8	....	....	....	....	....	....	0	0.1	−0.1	0	1.1	−1.1
1.9	....	....	....	....	....	....	0	0.0	0.0	3	0.5	+2.5
2.0	....	....	....	....	....	....	....	....	....	0	0.2	−0.2
2.1	....	....	....	....	....	....	....	....	....	0	0.1	−0.1
+2.2												

The distributions of x_1 and x_2 are shown graphically in Figures 1–4, the observed values, as points joined by straight lines; the values computed from the distribution of absolute magnitudes and linear velocities, by continuous curves. The dotted curves represent the distributions when only the normal giants are included in the distribution of absolute magnitudes.

The distributions of absolute magnitudes for the two spectral groups are given in Table V and illustrated in Figures 5 and 6, the total number of stars being reduced to 1000. The distribution of absolute magnitudes among the stars of types K0–K2 shows four distinct maxima in accordance with the result obtained in *Contribution* No. 396.[1] The number of stars in this spectral interval is so large that separate solutions from the τ- and υ-components could be made with fair accuracy; these are distinguished by plus signs and crosses in Figure 5. The absolute magnitudes of the dwarfs are those determined from the trigonometric parallaxes. The existence of the separate maxima is indicated by both the parallactic and the peculiar velocities. The relative proportion of stars in the four groups is 13.9, 78.3, 6.5, and 1.3 per cent; their mean absolute magnitudes are -2.5, $+0.3$, $+2.7$, and $+6.1$ mag., respectively.

The stars of types K3–K9 show frequency maxima at absolute magnitudes -4.5, -0.1, and $+6.7$, the percentage in each group being 7.1, 90.5, and 2.4, respectively.

The four groups among the early K stars were designated in *Contribution* No. 396[1] by the terms "supergiants," "ordinary giants," "subgiants," and "dwarfs." In that investigation, based exclusively on peculiar motions, the supergiants of the early K stars were of about the same brightness as those of late K type. The distribution of the brightest absolute magnitudes determined from parallactic motions has considerably higher weight than that based on peculiar motions. Further, the high galactic concentration of the supergiants introduces a systematic error into the absolute magnitudes derived from τ-components. These components are more or less perpendicular to the galactic circle, and the corresponding linear motion, on the basis of an ellipsoidal velocity distribution, averages less than that derived directly from the radial velocities. The absolute magnitudes

[1] *Ibid.*, **71**, 175, 1930.

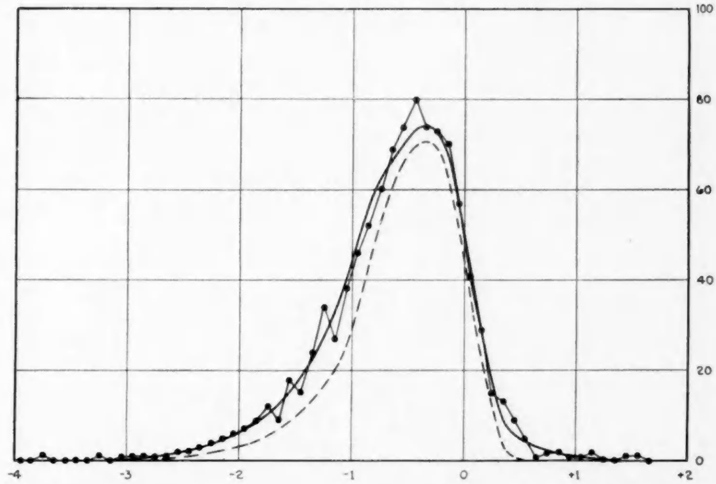

FIG. 1.—Distribution of $x_1 = \log \tau + 0.2\ m$ for stars of spectral types K0 to K2. The dots represent observed numbers within intervals $dx = 0.1$. The smooth curve is computed from the distributions of absolute magnitudes and peculiar radial velocities. The dotted curve shows the distribution when only the normal giants are included. For $x < -1.7$ the dots do not indicate independent observations, but show the probable extension of the distribution-curve.

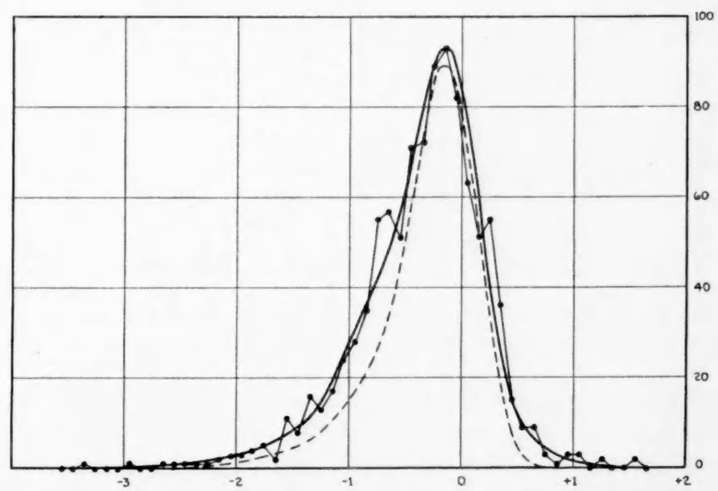

FIG. 2.—Distribution of $x_2 = \log v + 0.2\ m - \log \sin \lambda$ for stars of spectral types K0 to K2. Similar to Fig. 1, but the computed curve is based on a combination of group motion and peculiar radial velocities.

for the supergiants determined from τ-components are therefore systematically too faint. This explains why the present analysis makes the supergiants of late K type considerably brighter than did

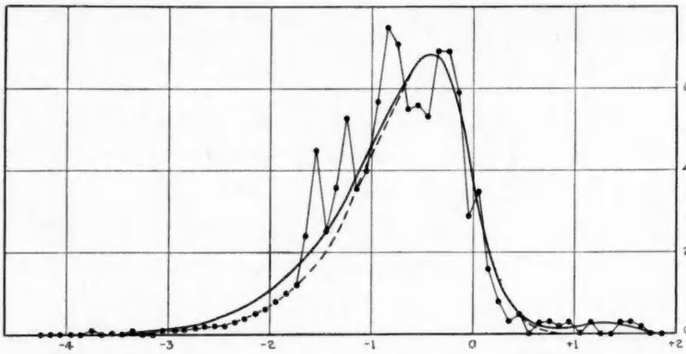

FIG. 3.—Distribution of $x_1 = \log \tau + 0.2\ m$ for stars of spectral types K3 to K9. Otherwise similar to Fig. 1.

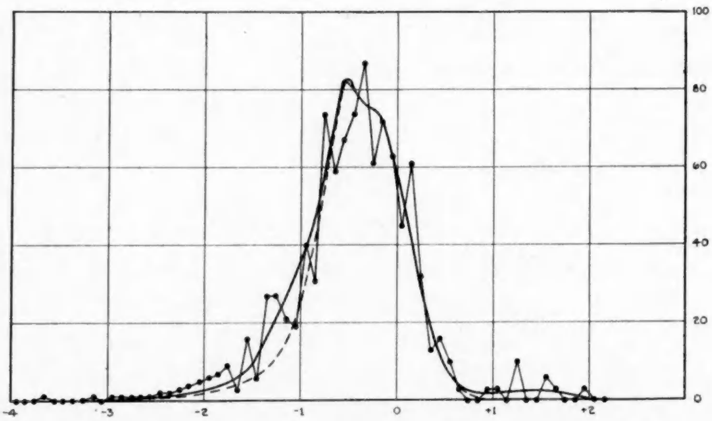

FIG. 4.—Distribution of $x_2 = \log v + 0.2\ m - \log \sin \lambda$ for stars of spectral types K3 to K9. Otherwise similar to Fig. 2.

the former investigation based on τ-components alone. It also explains certain systematic differences between the observed and computed values of the distribution $F(x_1)dx$ and $F(x_2)dx$ found for stars of types M and late K.

The present results indicate that the supergiants of late K type have about the same visual absolute magnitude as that found for

the supergiant M stars in *Contribution* No. 411.[1] It is thus quite possible that the group with a mean absolute magnitude −2.5 present among the early K stars is not an extension of the group of super-

TABLE V

$F(M)dM$

M	Ko–K2	K3–K9	M	Ko–K2	K3–K9
−5.8	0	1	+1.0	13	20
5.6	0	3	1.2	11	16
5.4	0	4	1.4	9	14
5.2	0	6	1.6	8	11
5.0	0	7	1.8	7	9
4.8	0	7	2.0	7	6
4.6	0	8	2.2	7	4
4.4	2	7	2.4	8	3
4.2	3	7	2.6	9	2
4.0	5	6	2.8	9	1
3.8	6	6	3.0	8	0
3.6	8	5	3.2	6	0
3.4	9	4	3.4	3	0
3.2	11	4	3.6	1	0
3.0	12	5	3.8	0	0
2.8	13	7	4.0	0	0
2.6	13	9	4.2	0	0
2.4	13	12	4.4	0	0
2.2	12	15	4.6	0	0
2.0	11	18	4.8	0	0
1.8	9	21	5.0	0	0
1.6	7	25	5.2	1	0
1.4	5	30	5.4	1	1
1.2	5	36	5.6	2	1
1.0	7	47	5.8	2	1
0.8	16	64	6.0	2	2
0.6	32	78	6.2	2	3
0.4	67	93	6.4	2	3
−0.2	100	98	6.6	1	4
0.0	135	87	6.8	0	3
+0.2	137	64	7.0	0	3
0.4	132	48	7.2	0	2
0.6	83	33	7.4	0	1
0.8	28	25	7.6	0	0
+1.0			+7.8		

giants of absolute magnitude −4.5 among the late K and the M stars. The probability that the groups are of different nature is strengthened by the fact that, although stars having *c*-characteristics, which are really the same as the pseudo-Cepheids, exist among both the late K and the G stars and are about absolute magnitude

[1] *Astrophysical Journal*, **72**, 117, 1930.

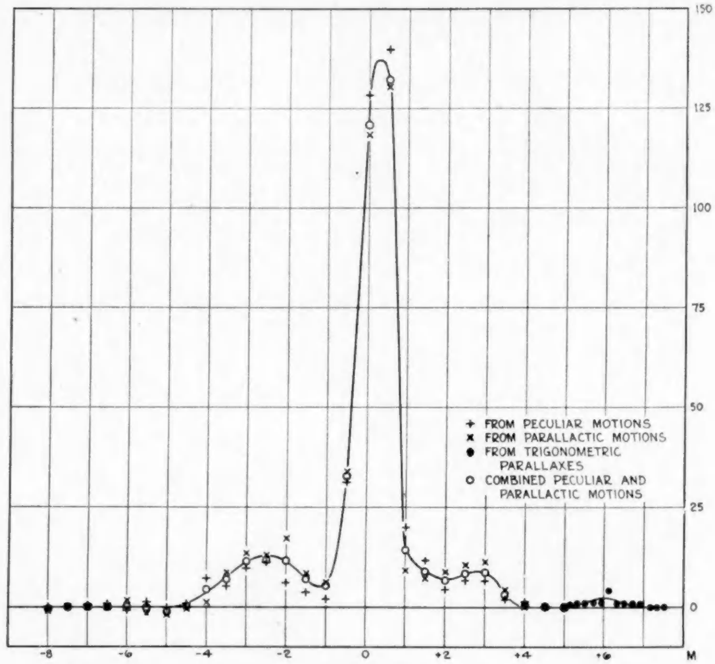

FIG. 5.—Distribution of absolute magnitudes for stars of spectral types K0 to K2 brighter than the sixth apparent magnitude.

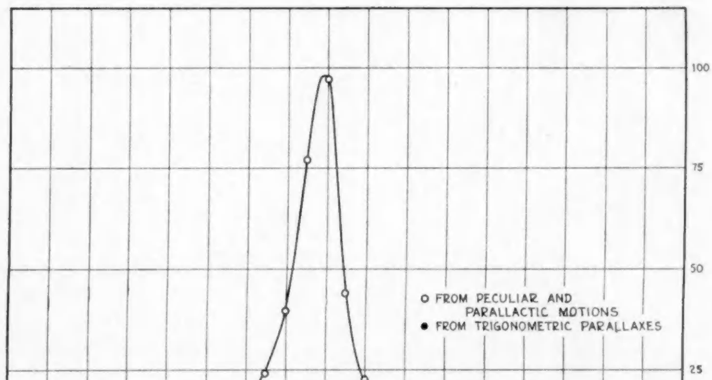

FIG. 6.—Distribution of absolute magnitudes for stars of spectral types K3 to K9 brighter than the sixth apparent magnitude.

-4, only one or two of them are found among the numerous stars in the spectral interval Ko–K2. If future investigation identifies the pseudo-Cepheids with the supergiants found by the statistical investigations, we shall have no right to use the term "supergiants" for the group of early K stars of a mean absolute magnitude -2.5. The name *bright giants* may be adopted provisionally. In conformity with this new notation the name *faint giants* may replace the name subgiants, the latter name having also the disadvantage, when abbreviated, of being difficult to distinguish from the term supergiants. Also the term *normal giants*, which has been used in the summary, will in future investigations be used to designate the large group of stars of type G to M and of absolute magnitude about zero.

A similar study of F and G stars will appear in a future *Contribution.*

Carnegie Institution of Washington
Mount Wilson Observatory
November 1930

MINOR CONTRIBUTIONS AND NOTES

AN ABNORMAL PHENOMENON OF PHOTOGRAPHIC PLATES

ABSTRACT

A reversal of low densities on certain photographic plates is noted, with possibly serious consequences to photometric and spectrographic exposures.

It is now quite customary for workers in astronomical photography to calibrate their plates by impressing on them a series of so-called sensitometric exposures. Normally, if such a series is printed on a plate, their densities will range from one just visible as a dark deposit against the surrounding fog to a deposit of a full blackness, in steps depending on the relative intensities adopted in the sensitometer. These latter usually range in intensity from the first to the fourth root of 2.

In recent tests of a standard emulsion—in fact, the emulsion in most common use at the present time for a great variety of astronomical work, including spectrographic and photometric—the usual sensitometric tests showed a reversal of the lowest intensities, their densities being less than the surrounding fog. This phenomenon, while not a new one, although comparatively rarely observed, has an important bearing on astronomical photography, and, in particular, on photometry. I am inclined to believe that this particular characteristic is present to a greater extent than would appear from rarity of its observation, the tests appropriate to its appearance not being made. Experiments were carried on with several developers and a number of emulsions. The effect was found for all developers only on the above-mentioned emulsion. With increase of time of development the reversal involved higher and higher densities. For a comparatively short development, giving a very clear background, the effect was not noticed. But for a normal time of development, such as is usually given by the writer in order to bring out everything there is on the plate, which means a certain amount of background fog, the reversal effect was quite pronounced. Under such

conditions, the effect on the apparent speed of the plate is exceedingly mischievous. The speed may be reduced from two to four times. It is strongly recommended that new emulsions be tested in the manner described above. Prolonged development should be given in order to increase the abnormal effect, if present. It is hardly necessary to enlarge on its evils. On account of the dip in the characteristic curve of an emulsion with this peculiarity, photometric measures are seriously impaired. It is conceivable that in the case of spectrograms faint emission lines would appear as absorption lines, and conversely absorption appear as emission lines.

The effect above described is of course not the ordinary reversal, which appears at high and not low densities. While apparently related to the Eberhard effect, it is difficult to see the exact connection on account of the low densities involved.

FRANK E. ROSS

YERKES OBSERVATORY
December 8, 1930

NOTE ON THE VARIABLE LINES OF HYDROGEN IN THE SPECTRUM OF 52 π AQUARII

ABSTRACT

In response to Mr. Higgs's request for some recent history of this star by other observers, the results secured by me from *photographs taken at the Norman Lockyer Observatory* at Sidmouth are here given.

The observations made here corroborate *the great change* which has taken place *in the spectrum from 1928 up to May, 1930*, as recorded by Higgs.

These observations however, which suggest a period for this star of approximately $1194 \pm$ days, cannot be reconciled to the estimates of relative intensity as recorded by him for the years 1915 and 1919.

In the October number of this *Journal* (**72**, 189) Mr. C. D. Higgs states that "it would be interesting to ascertain if the present type of change is a recent development of the star's history, or whether the observational record of the previous twenty-five years shows a similar behavior."

The spectrum of this star has been and is under observation at this, the Norman Lockyer Observatory at Sidmouth, and photo-

graphs were secured during the years 1923 and 1924 and also during the years 1928, 1929, and 1930. The reduction of these observations suggests that this star is undergoing a (possibly) regular variation with a period of about 1194± days, the rise to maximum being probably shorter than the fall to minimum, as deduced at present from a very approximate curve showing the differences of intensity of the violet and red components of the $H\beta$ line.

It appears that Higgs's scale of estimation of intensities is not quite the same as mine. In his Table I (on p. 189) he estimates the relative intensity of the red and violet components of $H\beta$ in September, 1928, as $R \rangle\rangle V$ (R much greater than V), while from my photographs taken during the same month my estimate is $R \rangle\rangle\rangle V$ (R very much greater than V).

Again, from the same table his estimate for May, 1930, is $V \rangle R$ (V greater than R), while my estimate from my curve is $V \rangle\rangle R$ (V much greater than R). My nearest observation to this month is July, in which my estimate is $V \rangle\rangle\rangle R$ (V very much greater than R), and this estimate holds also for the following months of August, September, and December in which I secured photographs.

There is little doubt that the intensity-curve has changed from a maximum ($R \rangle\rangle\rangle V$) in October, 1928, to a minimum (approximately; $V \rangle\rangle\rangle R$) in approximately October, 1930, and this great change is corroborated by Higgs.

My observations in 1923 and 1924 suggest a minimum about March, 1924, i.e., $V \rangle\rangle\rangle R$ on the scale of my method of estimation. From this minimum to that in October, 1930, is 2405 days, and as this covers two periods, the length of the period is about 1202 days. From two maxima I estimate the period as being 1187 days, and the mean of the two very approximate determinations is thus 1194± days as previously stated.

I cannot reconcile such quick changes of relative intensity as recorded by Higgs's Table I with my observations, such as the change from $R \rangle V$ to $R = V$ between the dates July 30 and August 6 in 1915, or the change from $R = V$ to $V \rangle R$ between October 31 and November 14 in 1919, or the reversion to $R = V$ again on November 17 of the same year.

The conclusion to be drawn, therefore, is that this star will have to be kept under very close observation in the future, and even daily observations will have to be made if such quick changes, as recorded by Higgs, are in progress.

My program has been to get monthly photographs, and so far my results are in good accord with a period of long duration.

WILLIAM J. S. LOCKYER

REVIEWS

Bandenspektren auf experimenteller Grundlage. By RICHARD RUEDY. Braunschweig: Friedr. Vieweg and Sohn Akt.-Ges., 1930. Pp. 124. Figs. 62. M. 9.60.

According to the author, this book is intended as an introduction to the analysis of band spectra for the research worker and student who attempts to unravel a new band. Although many reviews have appeared, mostly in German periodicals, dealing with various aspects of band spectra, this is the first since the publication of Professor Birge's chapter in the *Report on Molecular Spectra*, which treats of this particular subject in detail. It lacks—probably intentionally—a general treatment of the theory of molecular spectra, which in our opinion is necessary to those who may attempt to unravel a new band.

A large number of the better-known spectra of the simpler diatomic molecules are dealt with in detail, and these examples, together with the many diagrams included in the text, should be of great assistance to the research worker. There is, however, a scarcity of references, the list given in the last two pages being altogether inadequate. References to the sources from which examples have been drawn are given on the last page, but for only a few cases.

The nomenclature in band spectroscopy has been considerably revised since the publication of this book. In so far as the general reader is concerned this is not a detriment, but it may prove confusing to the student who uses this text as an introduction to the analysis of band spectra.

ANDREW CHRISTY

Life and Work of Sir Norman Lockyer. By T. MARY LOCKYER and WINIFRED L. LOCKYER, with the assistance of PROFESSOR H. DINGLE, and contributions by DR. CHARLES E. ST. JOHN, PROFESSOR MEGH NAD SAHA, SIR NAPIER SHAW, PROFESSOR H. N. RUSSELL, REV. J. GRIFFITH, SIR RICHARD GREGORY, and PROFESSOR A. FOWLER. Pp. xii+474. 17 plates. London: Macmillan & Co., Ltd., 1928. 18*s*. net.

There is a feeling when one puts down *The Life and Work of Sir Norman Lockyer* that the half has not been told us. Here is an orderly

account of an extraordinary man. It is a biography that could have been written with color and emphasis, but the picture that is drawn for us leaves us a bit weary and unsatisfied. The material was collected and collated by those who knew Sir Norman best, Lady Lockyer and his daughter Winifred Lockyer. The account was written by Herbert Dingle, professor of astrophysics at the Royal College of Science.

The book is divided into two parts. The first 266 pages are devoted to a general biographical sketch, and the remaining 196 pages to articles written by a group of distinguished authors on the work of Sir Norman as viewed in the light of our present knowledge. The division of the book is undoubtedly the chief cause of its lack of unity and emphasis. We would have caught a more intimate glimpse of the man if we could have read in the biography itself more about "the celestial dissociation of the elements" than to have had it "described at length in another chapter." A scientist's work is such an integral part of his life that a chronological account that does not picture him in the laboratory recording his successes and failures is bound to leave us with the impression that we are reading a list of events.

To those who read *Nature* the book is of especial value. It reveals to us the genius that was the guiding force of this great periodical for the first half-century of its existence. It is easy to account for the fact that today *Nature* is the leading weekly scientific publication. Lockyer's passion for acquiring knowledge, his ability to make new facts intelligible and attractive to others, his keen foresight, his power for organizing, his ability to inspire others, and his courage and faith made him a great editor.

It is an amazing career. For the most part Lockyer was self-taught. At the age of twenty-seven he obtained a temporary position in the war office. A year later, having passed a competitive examination, he was appointed to an established clerkship. His interest in astronomy and observing began in 1861 with the purchase of a $3\frac{1}{2}$-inch Cooke telescope. As the scientific editor of the *Reader*, a weekly "review of literature, science and art," in 1863–1865, we find the source of much of his inspiration. "There is little doubt that much of the work which he achieved later was, if not the direct development, at least the revival of germs of thought whose origin belonged to this period." The discovery of helium, the dissociation hypothesis, the meteoric hypothesis, theories concerning the constitution of the sun, the famous Lockyer-Janssen discovery, the interesting venture into the field of archaeology, the publication of book after book, one struggle after another as the champion of a cause that

seemed to him vital for the public welfare, planning expeditions, writing, lecturing, together with the dull routine for many years as a secretary in the war office—all went to make up the life and thought of this man. It is an astounding account of energy and productivity. One rather suspects that Sir Norman had a rather high opinion of himself and of what he could do, that he was not always as considerate as he might have been, that not all of his ideas were of the "first water"; but we have, in astronomy and astrophysics, much that has come to us as a result of his activity.

The chapters in the second part of the book which are devoted to special subjects such as "The Constitution of the Sun," "The Dissociation Hypothesis," "Dissociation Equilibrium," etc., contain no new material. They are interesting because they show Lockyer's influence in modern astronomy, astrophysics, and archaeology.

CLIFFORD C. CRUMP

The Physical Principles of the Quantum Theory. By WERNER HEISENBERG. Translated into English by CARL ECKART and FRANK C. HOYT. Chicago: University of Chicago Press, 1930. Pp. xii+186. $2.00.

This little book by the founder of quantum mechanics gives a beautifully clear account of that part of the theory which is concerned with the explanation of the wave-particle dilemma in atomic physics in terms of the "uncertainty principle." An introductory chapter gives a brief presentation of the basic experiments: Wilson photographs, electron diffraction, X-ray diffraction, Compton-Simon experiment, and the Frank and Hertz critical potentials.

The second and third chapters are devoted to critiques of the corpuscular and wave theories with very full discussions of the limitations imposed by the fundamental uncertainties of measurement in the field of atomic physics. In the third chapter there is a noteworthy presentation of the leading ideas with which attempts have been made to find a quantum mechanical revision of classical electrodynamics.

The fourth and fifth chapters present, respectively, the principles of the statistical interpretation of the theory and a detailed discussion of the application of the principles to the basic experiments. An Appendix gives a rather fuller account of the mathematical apparatus of the quantum theory, based on Heisenberg's Chicago lectures of the spring of 1929, than is given in the German edition.

E. U. CONDON

PRINCETON UNIVERSITY